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Preface

This book is a monograph from submissions from the 6th International Conference on Fuzzy Information and Engineering (ICFIE2012) on October 25–26, 2012 in Babolsar, Iran and from the 6th academic conference from Fuzzy Information & Engineering Branch of Operation Research Society of China (FIEBORSC2012) during December 18–24, 2012 in Shenzhen, China. The monograph is published by *Advances in Intelligent Systems and Computing* (AISC), Springer, ISSN: 2194-5357.

This year, we have received more than 300 submissions. Each paper has undergone a rigorous review process. Only high-quality papers are included in it. The book, containing papers, is divided into Seven main parts:

Part I—themes on "Programming and Optimization".
Part II—subjects on "Lattice and Measures".
Part III—topics are discussed on "Algebras and Equation" appearance.
Part IV—ideas circle around "Forecasting, Clustering and Recognition".
Part V—focuses on "Systems and Algorithm".
Part VI—thesis on "Graph and Network".
Part VII—dissertations on "Others".

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We wish to express our heart-felt appreciation to the Editorial Committee, reviewers, and our students. In particular, we are thankful to Doctoral: Ren-jie Hu, Hong Mai; Master: Zhi-ping Zhu, who have contributed a lot to the development of this issue. We appreciate all the authors and participants for their great contributions that made these conferences possible and all the hard work worthwhile. Meanwhile, we are thankful to China Education and Research Foundation; China Science and Education Publishing House, and China Charity Press Publishing and its president Mr. Dong-cai Lai for sponsoring.

Finally, we thank the publisher, Springer, for publishing the AISC (Notes: Our series of conference proceedings by Springer, like *Advances in Soft Computing* (ASC), AISC, (ASC 40, ASC 54, AISC 62, AISC 78, AISC 82 and AISC 147, have been included into EI and all are indexed by Thomson Reuters Conference Proceedings Citation Index (ISTP)), and thank the supports coming from international magazine *Fuzzy Information and Engineering* by Springer.

March, 2013

Bing-yuan Cao
Hadi Nasseri

Contents

Part VI Graph and Network

Part VII Others

Part I
Programming and Optimization

Fuzzy Modeling of Optimal Initial Drug Prescription

Mostafa Karimpour, Ali Vahidian Kamyad and Mohsen Foroughipour

Abstract This paper focused on a fuzzy approach in migraineurs drug prescription. There is no denying that medicine data records are mixed with uncertainty and probability so all methods concerning migraine drug prescription should be considered in fuzzy environment and designed according to the knowledge of an expert specialist. Overall it seems logical to propose fuzzy approach which could cover all uncertainties. According to fuzzy rule base concepts which are obtained by co-operation of an expert, fuzzy control has been used to model drug prescription system. Finally clinical experiences are used to confirm efficacy of drug prescription model. It should be considered that in most cases this disease may cause health social problems as well as financial obstacle for the companies and in upper stage for governments as a result of reduce work time among the employees. So prescribing optimal initial drug to make the disease stable is obligatory. With the corporation of experts 25 rule bases are introduced and tested on 50 different patient records, result shows that the accuracy of model is 94 % $\pm$ 5.5 with r-square equal to 0.9148.

Keywords Fuzzy control · Mamdani's inference approach · Migraine drug prescription

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1 Introduction

Migraine is a French term derived from Hemi crania, a Greek word that means one side of the head [1]. But in practical migraine is a common type of headache usually felt as a severe unilateral throbbing headache that lasts 4–72 h and sometimes accompanied with nausea, vomiting and other symptoms [2]. Approximately 13 billion dollars in USA is lost as a result of reduced work productivity in migrainures and nearly 27 billion euros in European community yearly [3]. Research shows that only 56 % of people suffering migraine know that they have migraine [4].

International classification of headache disorders 2nd (ICHD) presented all types of headaches and migraines with different criteria for diagnosing each one [5]. Medical diagnosing could be done in a variety of classification methods to name a few one could refer to the following: artificial neural networks, fuzzy control, support vector machines, etc. Previously some research is done in migraine diagnosing and migraine state to state rate of transition with drug. Mendes et al. [6] used artificial neural networks to classify headache into four different classes as tension type headache, medication-overuse, migraine with aura and migraine without aura. Simić et al. [3] Classified different types of migraine by employing rule-based fuzzy logic, which is suitable for knowledge-based decision making.

Maas et al. [7] proposed markov models as a powerful way of nonlinear system modeling in migraine drug response modeling. To get more insight into the concentration-effect markov model is proposed to describe the course of headache response to either placebo or sumatriptan. Figure 1 shows the structure of the markov response model.

As it could be seen from Fig. 4 The model is consist of two layers: first, state layer (hidden layer) which represents three different states of migraine (no relief, relief, and pain free). Second score layer (open layer) which shows the headache score (no pain, mild pain, moderate pain and severe pain).

The mentioned model is able to predict headache states and headache recurrence in migraine patients who receive placebo or oral sumatriptan. Scientist showed that sumtriptans are very effective in reducing migraine attacks. Oral doses of 50 or

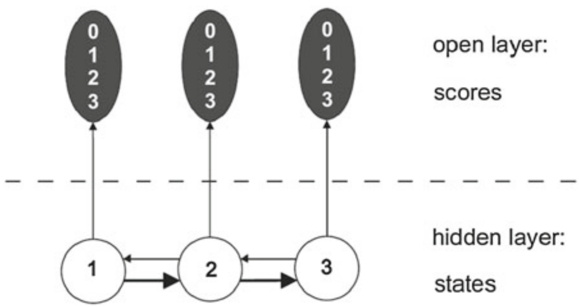

Fig. 1 Structure of a markov model

100 mg are usually prescribed to treat migraine attacks. The impression of sumatriptan on forward transition rates was expected to follow (1).

$$Rate(t)_{x,y} = Rate(0)_{x,y} \cdot exp\left(\frac{Emax_{x,y} \cdot C(t)}{EC50_{x,y} + C(t)}\right) \tag{1}$$

In Eq. 3 $Rate(t)_{x,y}$ shows the rate of drug-induced in forward transitions, $Rate(0)_{x,y}$ is the transmission rate from state x to y in the absence of sumtriptan. $C(t)$ is the sumtriptan plasma concentration. $Emax_{x,y}$ reveals the maximum effect of sumatriptan on forward transition rate. $EC50_{x,y}$ is the sumatriptan concentration related to half of the maximum effect. Anisimov et al. [8] showed that if x>y

$$Rate(t)_{x,y} = Rate(0)_{x,y} \tag{2}$$

Equations (1) and (2) were chosen based on the fact that the increase dose of sumtriptan drug leads to the headache pain decrease. In this theory this observation can be modeled as (a) Increasing forward transition rate is a function of drug concentration. (b) Reducing backward transition rate is a function of drug concentration.

In migraine, headache severity depends on age so the effectiveness of medication may also depend on age. Maas et al. [9] parameterized the interaction between age and drug exposure. The proposed model also reveals the clinical observations such as: (a) The rate which changes state pain relief to pain-free relates to age inversely. (b) In placebo-treatment, the mean transition time from 'no relief' to 'relief' is 3 h for young adolescents and 6 h for patients older than 30 years old. (c) Sumatriptan reduces the transition time to 2 h, without considering age. In this case by considering age Eq. (1) could be improved to Eq. (3).

$$\begin{aligned} Rate(t)_{x,y} &= \mathrm{Rate}_{\min} + \frac{Rate_{\max} - Rate_{\min}}{1 + f(age, C(t))} \\ f(age, C(t)) &= E_0 \cdot \exp\left(\frac{E_{\max_{age}} \cdot \exp(age))}{\exp(E50_{age} + \exp(age)} - \frac{E_{\max_{C(t)}} \cdot \exp(C(t))}{\exp(E50_{C(t)}) + \exp(C(t))}\right) \end{aligned} \tag{3}$$

$\mathrm{Rate}_{\min}$ and $\mathrm{Rate}_{\max}$ represents the minimum and maximum values of transition rate. f is the function that describes the association between patient age and sumatriptan concentration (c(t)) to the transition rate. At t=0 $\exp(C(t))$ must be replaced with zero.

Gradual titration of drugs used for treatment of migraine (like sodium valproate) takes a long time to reach the best dosage and also starting with high range dosages increases the side effect of the drug so choosing the best initial dosage plays an important role in migraine treatment. In this paper a novel fuzzy modelling method is proposed in which the best dosage of initial drug could be detected.

2 Methods and Materials

Sodium valproate is the sodium salt of valproic acid. Sodium valproate can be used to control migraine. In migraine sodium valproate drug prescription depends on different factors but the most important factors are 1- pain intensity, 2- pain frequency and 3- weight. So in this paper fuzzy rule bases are proposed through the help of an expert specialist to find optimal initial sodium valproate drug prescription in migraineurs to cure disease in the minimum time by considering drug side effects as model constraints. A fuzzy questionnaire should be complete with these parameters through the patients. This study is done in neurology ward of Ghaem hospital in mashhad from Mar 2011 to Aug 2012.

The related questions are presented in Table 1.

There are some basic hypothesis used on this research which should be considered in sodium valproate prescription system, presented as follows:

1. Patient should be diagnosed and detected as migrainures.
2. There are some restrictions such as low blood pressure, depression, drugs side effects which is better for the patient to use sodium valproate.

Membership function of pain intensity and pain frequency are proposed as follows:

1. Pain intensity which is measured through using virtual analogue scaling (VAS), can be described well with the following membership function shown in Fig. 2.

Table 1 Questioner

1- The intensity of pain
The answer referring to this question could be in the form of an integer number between 1 (low pain) to 10 (The maximum imaginable pain) for parameters
2- Frequency of pain during one month. An integer number between 0 and 10 (upper than 10 is considered as 10)
3- Patient's weight (kg)

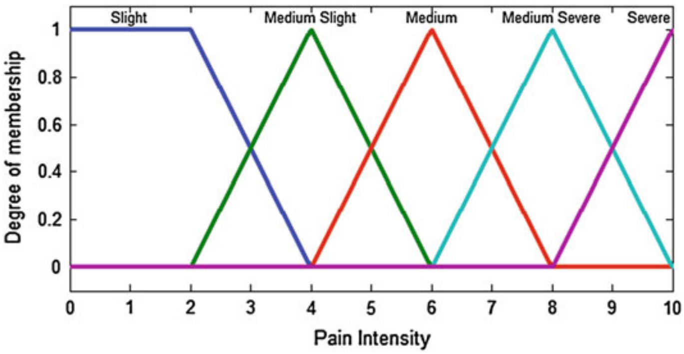

Fig. 2 Membership function of pain intensity

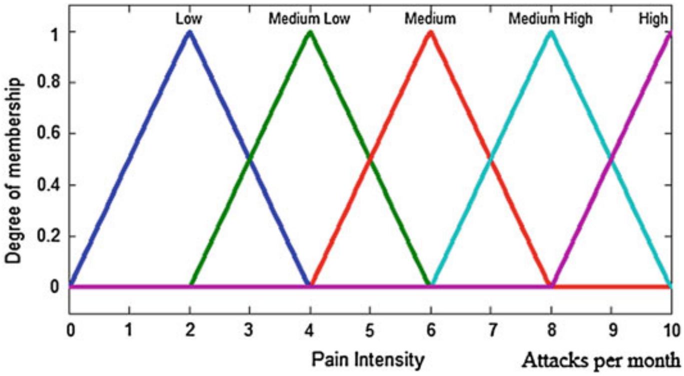

Fig. 3 Membership function of attack frequency

At the first part as the pain intensity is not detectable correctly and declared with the patient's feeling, so it is considered as trapezoidal membership function for slight in our approach, and a triangular membership function for the rest. It is divided into five parts 'Slight, Medium Slight, Medium, Medium Severe, Severe' to get the optimal decision making.

2. The frequency of migraine attacks which is presented in the form of number of attack per month is well presented with the chart provided in Fig. 3.

Pain frequency is completely measurable so all the membership functions are considered as triangular. Pain frequency is also divided into five classes 'Low, Medium Low, Medium, Medium High, High' which is really helpful in final decision making. It should be considered that increasing these classes increase mathematical computation and curse of dimensionality. Attack frequency more than 10 times per month, is considered as 'High'.

The consequent part of each rule is the membership function of sodium valproate dosage, the membership function is considered as Fig. 4.

Mamdani's fuzzy inference system is employed in sodium valproate optimal dosage calculation. As there are 2 different inputs with 5 different membership functions assumed, 25 different rule bases are introduced. Fuzzy rule bases are proposes through the consultation of expert specialists. The experimental results at the final part of this paper proof that the fuzzy control result with the comparison to reality (based on patient's records) is acceptable.

Fuzzy rule bases are presented in Table 2.

In real systems as the partition of uncertainty increases the efficacy of fuzzy logic rule bases increases. For complex systems usually knowledge exists and just few numerical data is available.

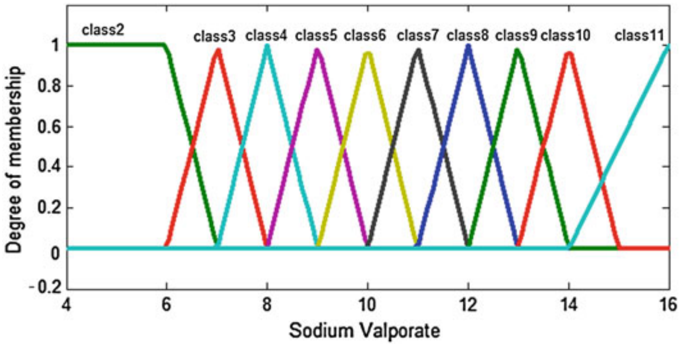

Fig. 4 Membership function of sodium valproate

Table 2 Rules developed for two inputs

Premises		Consequent
Paint intensity	Pain frequency	Sodium valproate
Severe	High	Class 11
Severe	Medium high	Class 10
Severe	Medium	Class 09
Severe	Medium low	Class 08
Severe	Low	Class 01
Medium severe	High	Class 11
Medium severe	Medium high	Class 10
Medium severe	Medium	Class 09
Medium severe	Medium low	Class 04
Medium severe	Low	Class 01
Medium	High	Class 10
Medium	Medium high	Class 08
Medium	Medium	Class 07
Medium	Medium low	Class 04
Medium	Low	Class 01
Medium slight	High	Class 04
Medium slight	Medium high	Class 03
Medium slight	Medium	Class 03
Medium slight	Medium low	Class 02
Medium slight	Low	Class 01
Slight	High	Class 01
Slight	Medium high	Class 01
Slight	Medium	Class 01
Slight	Medium low	Class 01
Slight	Low	Class 01

3 Results

Mamdani introduced a fuzzy inference system and developed a strategy which is usually referred to as max-min method. Mamdani's fuzzy inference system is a way of linking linguistic inputs to the linguistic outputs with just using min and max functions and allows gaining approximate reasoning [10]. Figure 5 represents model flow chart of our approach.

50 different patients who recursed to Ghaem hospital of Mashhad were inserted to the system. Data were gathered from Mar 2011 to Aug 2012 and were tested according to the rule bases presented in part 2. The experimental results are as presented in Table 3.

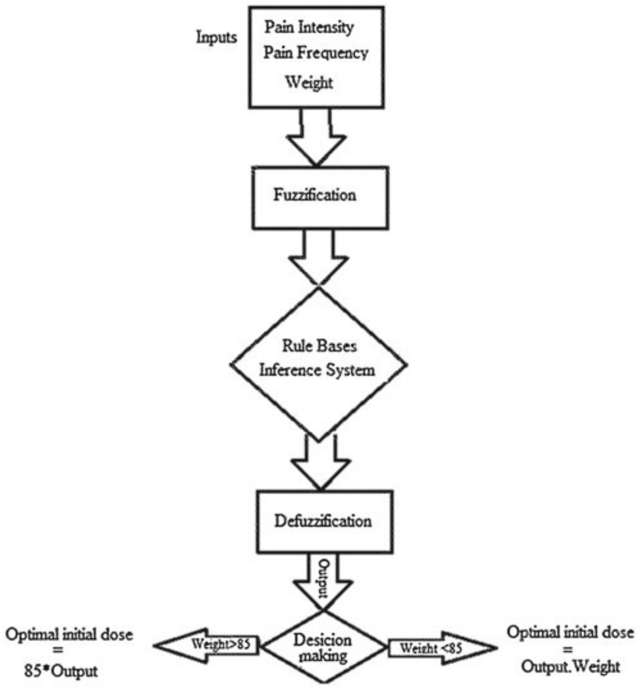

Fig. 5 Model block diagram

Table 3 Experimental results

Parameters	Test data
APE	5.9916
R square	0.9148
MSE	5229
RMSE	72.3561
Residual mean	44.6517

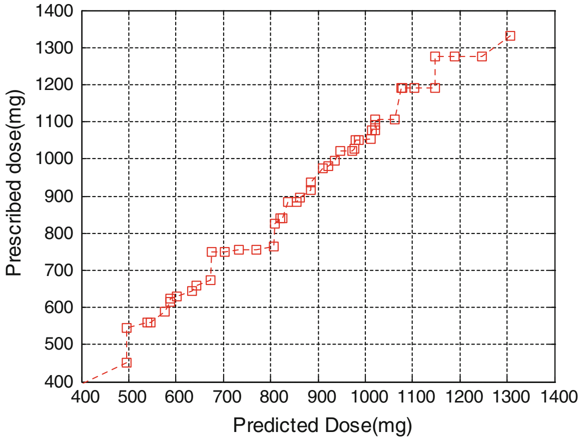

Fig. 6 Predicted and prescribed dosage of sodium valproate

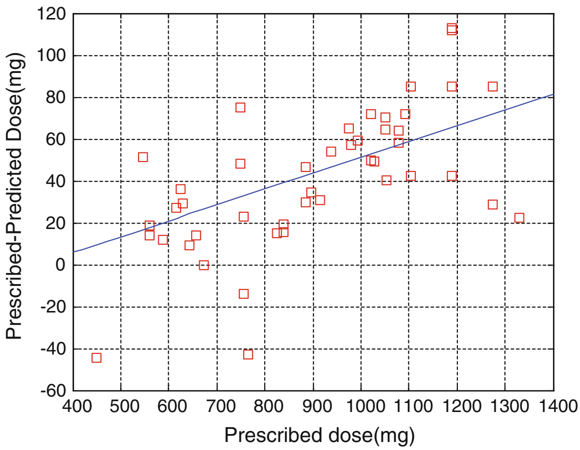

Fig. 7 Plot of prescribed minus predicted dosage per prescribed dosage

Result shows that the system is able to predict the optimal initial dose with an acceptable error. Data are taken from 50 patients who used certain initial dosage and pain frequency is reduced and intensity of their pain is relieved.

Figure 6 shows predicted and prescribed dosage of sodium valproate. Figure 7 presents plot of prescribed dosage minus predicted dosage per prescribed dosage of sodium valproate.

4 Conclusion

Defining the patterns of migraine acute treatment in the population is an important step in evaluating migraine treatment in relation to guiding principles, and improving health care [11].

Research shows that migraine headaches are not often treated effectively in many people who suffer from them. Preventive treatments are effective in about 38 % of people suffering migraine. Less than one-third of those people currently use these types of treatments. Fortunately, in many people the frequency and severity of migraine attacks which are the important factors of this study can be reduced with preventive treatment. In fact, some studies show that migraine attacks may be reduced more than half. Some epilepsy drugs are useful in preventing migraine. Strong evidence shows divalproex sodium, sodium valproate, and topiramate are helpful in preventing migraine [12].

Research shows that extended-release divalproex sodium 500–1,000 mg per day had an average reduction in 1 month migraine headache prevalence with a rate (-1.2 attacks per week) from 4.4 per week (baseline) to 3.2 per week. In most headache trials, patients taking divalproex sodium or sodium valproate reported no adverse events [13]. According to what has been mentioned one of the most effective and most widely used drugs in treating migraine is sodium valproate, but there are two factors that force the optimal usage of this drug: the side effect and the cost [14] but in Iran side effect is much more important because of the insurance cover on drug price.

Gradual titration of sodium valproate is one way of avoiding side effects but it takes a long time to reach the best dose. Recommended daily dosage ranges from 800 to 1,500 mg [15]. In Iran the start point could be 400 mg or lower [16].

In this paper for the first time a new method in optimal migraine drug prescription system is proposed. Mamdani fuzzy inference system is developed in predicting optimal sodium valproate initial dosage. Gradual titration of sodium valproate is one way of avoiding side effects but it takes a long time to reach the best dosage. So a mathematical model is proposed through the knowledge-driven from expert specialists. Simulation result indicates that model is able to predict sodium valproate optimal initial dosage with the accuracy of 94 % ± 5.5.

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Decision Parameter Optimization of Beam Pumping Unit Based on BP Networks Model

Xiao-hua Gu, Zhi-qiang Liao, Sheng Hu, Jun Yi and Tai-fu Li

Abstract Beam pumping unit is the most popular oil recovery equipment. One of the most common problems of beam pumping unit is its high energy consumption due to its low system efficiency. The main objective of this study is modeling and optimization a beam pumping unit using Artificial Neural Network (ANN). Among the various networks and architectures, multilayer feed-forward neural network with Back Propagation (BP) training algorithm was found as the best model for the plant. In the next step of study, optimization is performed to identify the sets of optimum operating parameters by Strength Pareto Evolutionary Algorithm-2 (SPEA2) strategy to maximize the oil yield as well as minimize the electric power consumption. Forty-nine sets of optimum conditions are found in our experiments.

Keywords Modeling · Artificial neural network · Optimization · Strength pareto evolutionary algorithm-2 · Beam pumping unit

1 Introduction

Beam pumping unit (BPU) is one of most important oil recovery equipments, the occupancy of which reaches to 70 % [1]. However, because of the negative torque, long gear train, poor working conditions and other reasons, the system efficiency of

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BPU is very low. So the research for energy saving of BPU is very important and necessary [1, 2].

Currently, methods of BPU energy saving are mainly mechanical ones and electrical ones [3]. In the first aspect, researchers focus on changing the structure of the pumping unit, adjusting the balance of the pumping unit and so on [4]. Whereas, electrical methods attempt to improve the motor control technology [5, 6]. In recent years, intelligent algorithm used to BPU energy saving is a new trend [7]. In this paper, we aim at building up an ANN for simulating the BPU, and then searching the optimum operation parameters based on the trained model.

The process of BPU is very complicated in nature typically due to unknown dynamic behaviors, nonlinear relations and numerous involved variables. Developing an accurate mathematical model are very hard even impossible. ANN modeling is a new choice to manage the complexities mentioned since it only requires the input-output data as opposed to a detailed knowledge of a system [8]. To identify optimum operating parameters, optimization based on ANN model is also performed. In this case, we consider to minimizing the electric power consumption and maximizing the oil yield, which leads to a multi-objective problem (MOP). SPEA2 [9], which has characteristics of stability, global search capability, nearest neighbor density estimation and new truncation method, is an ideal choice. Consequently, in this paper, BP networks is employed to model the BPU and SPEA2 is applied to the trained BP network model to determine the parameter values to energy saving and yield increasing.

2 Method Study

2.1 BP Networks

Multilayer feed-forward neural network with Back Propagation (BP) training algorithm is proposed by Rumelhart and Mcclalland [10] to solve the multi-layer network learning algorithm problem in 1986. It consists of input layer, hidden layer, and output layer. In BP networks training stage, the error between the experimental data and the corresponding predicted data is counted and back-propagated through the network. The algorithm adjusts the weights in each successive layer to reduce the error. This procedure is circulated until the error between the experimental data and the corresponding predicted data satiety certain error criterion. BPN models compute the output value as a sum of non-linear transformations of linear combinations of the inputs. The data predicted by BPN models were plotted against the corresponding experimental data to visualize the modeling abilities of the BPN models.

2.2 SPEA2

SPEA2 which proposed by Zitzler [9] in 2001 is a multi-objective evolutionary algorithm characterized by the concepts of strength and density. SPEA2 is an extension of SPEA, it has an improved fitness assignment strategy and hence can search the global optimum of all objective functions. Consequently, SPEA2 becomes one of the most popular optimization techniques in multi-objective combinatorial optimization problems, as well as nonlinear ones.

3 BPU Modeling and Optimization Based on BPN-SPEA2

3.1 BPU

Beam pumping unit production system includes two parts: the ground part and the underground part. The mainly ground devices include the dynamic force and the balance of beam-pumping unit, while the underground part includes the oil pump and the corresponding valve. With the in-depth study of the process, number of punching as the decision parameter is proposed, which updates the optimum number of punching that could response the change of status. Running in optimum number of punching always saves more energy. But just number of punching as input parameter to model of beam pumping unit production system is hardly approximation real model as the amount information is lacking. In order to solve this problem, the environment parameters are supplied as the input parameter to model. So the input parameters include number of punching(NP), maximum load(MAXL), minimum load(MINL), effective stroke(ES) and computational pump efficiency(CPE), while the output parameters are electric power consumption(EPC) and oil yield(OY). All the inputs and outputs of BP networks are show in Table 1.

3.2 BPU Modeling Based on BPN

BP networks algorithm is developed to model BPU. As shown in Fig. 1, the network is three-layer. The input neurons node in the network is five and output neurons node is two. Hidden layer's neuron number t is determined by an empirical formula $t = \sqrt{n+m} + \alpha$, where n is the number of input layer neurons, m is the number of output layer neurons, and α is a constant between 1 and 10.

The training was executed systematically with different number of nodes in hidden layer. Tan-sigmoid transfer function was used as the activation function for the hidden layers, and linear transfer function was used for the output layer. The values of the test data were normalized to within the range from -1 to 1.

Table 1 Input/output parameters of BP neural networks

Inputs		Outputs
Decision parameter	Number of punching (time/min)	Electric power consumption (kw/h)
	Maximum load (kN)	
Environment parameters	Minimum load (kN)	
	Effective stroke (m)	Oil yield (t/d)
	Computational pump efficiency (%)	

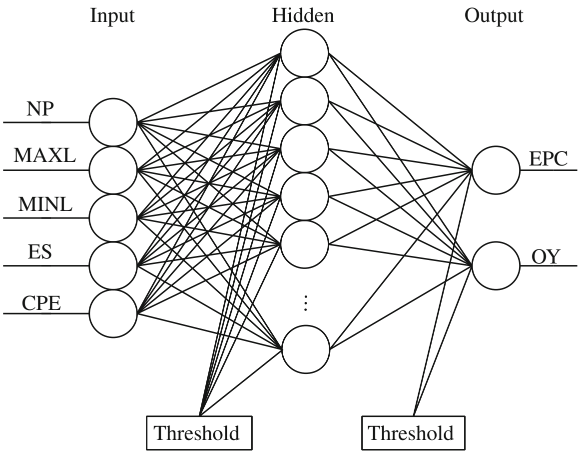

Fig. 1 BPN model of beam pumping unit production system

3.3 SPEA2 Optimization of Decision Parameter of BPU

Once the beam pumping unit production system process model based on BP networks is developed, it can be used to determine its fitness value for optimization to obtain the optimal values of the input variables that minimizing electric power consumption and maximizing oil yield. Considering that SPEA2 always searches the minimize fitness, hence the maximum objective is converted to minimum one by taking its negative. Finally, the beam pumping system energy saving multi-objective problem with 5 variables, 2 objectives is described as follow:

$$\hat{y} = \min F(\hat{x}) = \min(f_1(\hat{x}),\ f_2(\hat{x})) \tag{1}$$

In which, $\hat{x} = (x_1, x_2, \cdots, x_n) \in X$, $X = \big\{(x_1, x_2, \cdots, x_n)|l_i \le x_i \le u_i, i = 1, 2, \cdots 5\big\}$, $L = (l_1, l_2, \cdots, l_5)$, $\hat{y} = (y_1, y_2, \cdots, y_n) \in Y$, $U = (u_1, u_2, \cdots, u_5)$, where L is the lower boundary of optimal parameters and U is the upper boundary of optimal parameters.

Obviously, in above model, the electric power consumption and the oil yield are the two objectives of SPEA2, which means that the decision parameters are optimization to response the change status.

4 Experimental Results and Analysis

4.1 The Training and Simulation of BP Networks

To build up a BP network for simulation BPU, necessary data is provided. Among the available 3,234 data from DaGang oil field, three examples of the samples are

Table 2 Examples of samples

NO.	NP (time/min)	MAXL (kN)	MINL (kN)	ES (m)	CPE (%)	EPC (kw/h)	OY (t/d)
1	3.01	97.2	44.7	3.074646	69.26006	11.97	31.05339
2	2.99	98.4	43.5	3.131836	71.01529	12.44	31.48944
…	…	…	…	…	…	…	…
3,234	2.99	98.7	43.4	3.027187	69.71369	12.23	30.47277

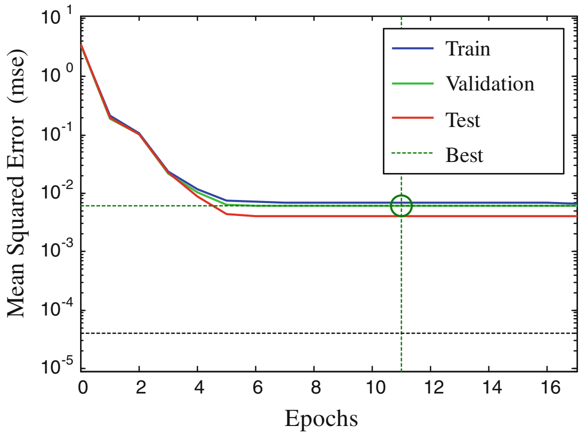

Fig. 2 The BP networks training process

shown in Table 2. The 3,234 samples are divided into two parts, 3,000 samples data are used to build up the model of BPU, while the other 234 are applied to test the generalization ability of trained model.

BP networks training was a supervisory learning process with cross validation. The Levenberg-Marquardt (LM) algorithm is repeatedly applied until the error threshold or the stop criterion is reached. Among many converged cases, the configuration 5-9-2 appeared the most optimal. Training started after setting BP network structure and parameters. When the BPN training precision meet the error limit expectation, as showed in Fig. 2, the training finished.

To evaluate the precision of the obtained model, the performance of the obtained model on the test set is tested. The comparisons of the predicted objectives and the real objectives are given in Fig. 3.

From the simulation results, we find that the predicted values and the real ones are very close. In other words the simulated values match well with the measured ones. It achieves a high prediction, and completely meets with the actual production needs. This confirms that the BPN based beam pumping process model is stable and reliable and could be regarded as a knowledge source for follow-up process parameters optimization.

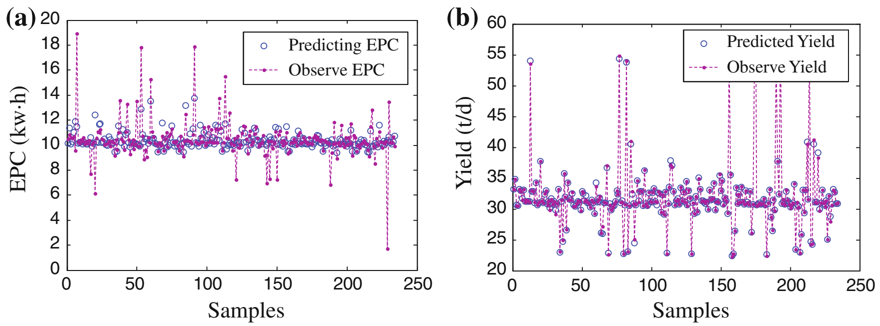

Fig. 3 The BP networks prediction results

Table 3 The limits of parameters used in optimization

Parameters	NP (time/min)	MAXL (kN)	MINL (kN)	ES (m)	CEP (%)
Limits	2.5–3.5	93.3–93.5	42.3–42.5	3.1–3.2	71–72

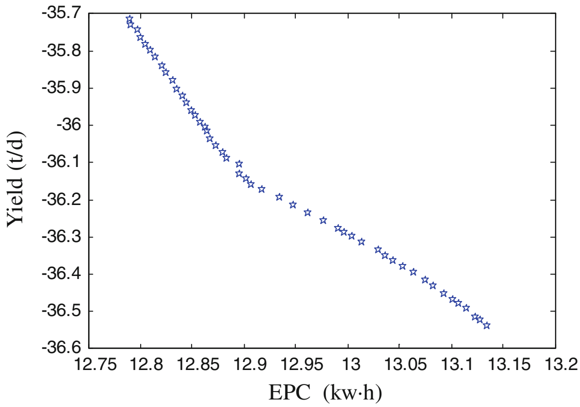

Fig. 4 The Pareto optimal front of SPEA2

4.2 Decision Parameter Optimization by SPEA2

Using the designed network, the effect of the operating parameters of BPU, like the number of punching, maximum load, minimum load, effective stroke, computational pump efficiency, on electric power consumption and oil yield are studied. Table 3 shows domain of change for input parameters.

In the SPEA2 algorithm, initial population, evolutionary generation, offspring individuals and parent individual are set to 80, 500, 40, and 40.The results of beam pumping unit Pareto frontier are shown in Fig. 4. Three instances of solutions of Pareto optimal set is shown in Table 4.

Table 4 The Pareto optimal front of SPEA2

NO.	NP	MAXL	MINL	ES	CPE	EPC	OY
1	3.5	93.4979	42.4996	3.10132	71.9995	12.90683	36.159
2	3.49999	93.3	42.4999	3.10004	71.7451	12.85295	35.973
…	…	…	…	…	…	…	…
49	3.49999	93.3104	42.4998	3.10047	71.7582	12.82161	35.740

As shown in Table 4, different value of number of punching from Pareto optimal set can be set according to corresponding status, the result shows that electric power consumption is decreased above 5 % and oil yield is increased above 6 %.

5 Conclusion

In this paper, a hybrid BPN-SPEA2 strategy is proposed to achieve the optimum decision parameters in the beam pumping unit production system for the aim of saving energy. BPN is applied to build up the model of beam pumping unit, and decision parameters are optimization by SPEA2 based on the trained BPN model. The experiments are conducted on 3,234 real samples from DaGang oil field. The results show that the system performance using the optimum parameters is improved significantly. Specifically the electric power consumption decreases more than 5 % and oil yield increases more than 6 %. It provided the proposed method is a alternative effective solution for energy saving of oil field.

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Optimization Strategy Based on Immune Mechanism for Controller Parameters

Xian-kun Tan, Chao Xiao and Ren-ming Deng

Abstract Control quality of the controller depends on correct tuning of control parameter, and it is directly related to the control effect of whole control system. Aimed at the puzzle that the parameter of controller has been difficult to tune, the paper proposed a sort of optimization model based on immune mechanism for tuning of controller parameters. Firstly it defined the antibody, antigen and affinity of tuning parameter, and secondly explored the process of parameter tuning based on immune mechanism in detail, then explained the tuning method by means of optimizing parameters of PID controller as well as the seven parameters of HSIC controller. Finally it took a high-order process control as the example, and made the simulation to a large time delay process, and to a highly non-minimum phase process as well as HSIC controller. The simulation experiment results demonstrated that it is better in comparison with some other tuning methods for dynamic and steady performance. The research result shows that the proposed method is more effective for controller parameter tuning.

Keywords Immune mechanism · Genetic algorithm · Clone selection · Mutation · Parameter tuning

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1 Introduction

For recently several decade years,although lots of advanced control algorithms have been continually presented, but until nowadays, the conventional control algorithm of PID is always applied in process industrial control field widely, and also it has still taken the most part in the actual engineering applications. The key puzzle in actual engineering of industrial control fields is very difficult to tune the parameters of PID controller so as to influence the control quality being not so good. Therefore it is necessary to research the method of tuning parameter for PID controller. According to the partition of development stage, it can be divided as the routine and intelligent tuning method of PID parameters [1]. In terms of the partition of controlled object number, it is generally divided as single-variable and multi-variable tuning method of PID parameters. By means of combination form of control quantity, also it can be divided as linear and non-linear parameter tuning method of PID controller, the former is used to the classical PID controller, and the latter is used to the nonlinear PID controller created by combination mode of non-linear track-differentiator and non-linear combination. With the rapid development of intelligent control technology, in order to improve the performance of PID controller, it has been come forth to lots of intelligent PID controller. Those controllers fused the multi-aspect technologies such as fuzzy control [2], NN [3], ant algorithm [4], genetic arithmetic [5, 6] and so on. It made PID controller own the self-learning ability and therefore extended the application bound of PID controller. Based on the enlightening of biology immunity system, the paper proposed a sort of parameter tuning model of controller based on the immune mechanism.

2 Process of Parameter Tuning for Controller

Due to the classical control algorithm of PID being conventional tuning manner, in order to improve the system performance, for convenience, here it takes the PID parameter tuning as an example, and makes the anatomy explain why it is the puzzle. Therefore it is necessary to research the influence on control system characteristic for each parameter in the control algorithm of PID. The following would respectively explain the influence on dynamic and steady performance of the whole system for each node parameter of PID.

1) The action of proportional node is to reduce the deviation of system. If the proportional coefficient KP is enlarged then it would be expedited to the response speed, and reduced to the steady error and enhanced to the control precision for the whole system. But too large KP always results in overshoot, and leads to the system being instable. And if KP is too small then the overshoot of the whole system would be reduced so as to enlarge the steady-state abundant extent. But the system precision would be reduced, and it would make the system precision be reduced so as to result in time delay of transition process.

2) The action of integral node is to eliminate steady-state deviation, and to reduce the system error. But it would make that the system response speed becomes slow, the system overshoot gets large, and the system results in bringing oscillation. But it is propitious to reduce system error for increasing integral coefficient KI, however too strong integral action is able to make the system bring overshoot so as to bring system oscillation. Although reducing integral coefficient KI makes the system improve the stability, and avoids the system bringing oscillation, and reduces the system overshoot, but it is adverse for eliminating steady deviation.
3) The function of differential node can reflect the change trend of deviation signal. It is able to introduce a early modifying signal that it redounds to reduce system overshoot before deviation signal value going into too large. It redounds to reduce the system error, to overcome the oscillation, to make the system going to stable state fleetly, to enhance the system response speed, and to reduce the adjusting time, therefore the dynamic characteristic of system is improved. The disadvantages are poor in anti-jamming ability, and great in influence on process response. If the differential coefficient KD gets large then it is propitious to expedite the system response, to reduce the system overshoot, and to increase the system stability. But it would result in disturbance sensitive and weaken the restraining disturbance ability, and if the KD gets too large then it would result in response process advancing to apply the brake, and delay the adjusting time. And reversely, if the differential coefficient KD gets too small then the speed-down of system adjusting process would be delayed, and the system overshoot would be increased. It makes system response speed slow-down, and finally it would result in system stability being bad.

The above analysis shows that the tuning of PID parameter is very complicated, and even it is very difficult to make adjusting and controlling in the complicated system. It is the most difficult to select the three control parameters, KP, KI and KD correctly. Therefore it is necessary to seek the new method of parameter tuning so as to obtain the satisfactory control quality for whole system.

3 Description of Optimization for Control Parameter Tuning

Now consider a closed-loop negative feedback control system shown as in Fig. 1, in which, Gp(s) is the controlled object model, Gc(s) is the controller, r is the input of set value, e is the error signal, u is the controller output, y is the system output.

The aim of controller parameter tuning is to select a set of control parameter of PID controller for given controlled object by means of a sort of tuning method. Aimed at the given input, the tuning result of the system constituted by controlled object and controller is to make the one or several performance indexes obtain optimization under the condition of certain criteria. Therefore there are two main problems needed solving in optimization design of the controller, they are the selection of performance indexes and the choice of tuning method for seeking optimization parameters.

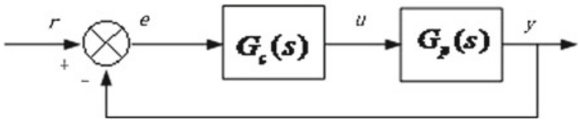

Fig. 1 Control system with closed-loop negative feedback

There are three aspects of indexes to weigh the control system performance. They are respectively the stability, precision and speediness. For example, the rising time reflects the system speediness, the shorter the rising time is, the sooner the control process response is, and the better the system control quality is.

If it only hankers for dynamic characteristic of system simply then it is very possible to make the gained parameter be too large for control signal. And therefore it would result in system being instable because of connatural saturation character in the system for actual engineering application. In order to get better control effect, it proposes that the control quantity, system error and rising time should have certain constraint condition. Because the accommodating function is related to the objective function, after determining the objective function, it could directly be as accommodating function to make the parameter seek the optimization value. The optimal parameter is the control parameter that is corresponding to x under the condition of satisfying constraints, and it makes the function f(x) to reach the extremum.

In order to obtain the satisfactory dynamic character of transition process, it adopted the performance index of error absolute time integral to be as selection parameter for least objective function. To avoid the control energy to be too large, it added a square item of control input in the objective function. The formula (1) is chosen as the optimal index of parameter selection.

$$J = \int_0^{\infty} (\omega_1 |e(t)| + \omega_2 u^2(t))dt + \omega_3 \times t_u \tag{1}$$

In which, e(t) is the system error, u(t) is the control output, t_u is the rising time, ω_1, ω_2 and ω_3 is respectively the different weighting value.

To avoid the system bringing overshoot, the castigation function is adopted. Namely, once the overshoot happens, the overshoot would be added an item of optimal performance indexes, here the optimal performance index would be expressed as formula (2).

$$If\ e(t) < 0, \quad J = \int_0^{\infty} (\omega_1 |e(t)| + \omega_2 u^2(t) + \omega_4 |e(t)|)dt + \omega_3 \times t_u \tag{2}$$

In which, ω_1, ω_2 and ω_3 and ω_4 is respectively the weight value, and $\omega_4 \gg \omega_1$.

Table 1 Mapping between human-body immune and immune based tuning model

Human-body immune system	Model of parameter tuning
Antigen	Optimal solution
Antibody	Feasible solution
Cell clone	Antibody copy
Binding of antibody and antigen	Value of antibody replacing the antigen
Cell B, Cell T	Vector
Increase of antibody density	Increase of approximate feasible solution

4 Design on Model Algorithm

In this paper, the proposed self-tuning algorithm of control parameter based on immune mechanism is similar to the working process of human body immunity system. The relationship between the human body immunity system and the proposed method of parameter tuning model is shown as in Table 1.

In the candidate solution generated by random, through the affinity computing of antigen and antibody, the superiority antibody is voted in, they hold the superiority gene, and make the mutation in the clone process and therefore lots of new antibodies are produced. Then it can reappraise for new antibody set and the new brought antibody also renews the antibody set. The updating mechanism of antibody is that the newly brought antibody, owned higher affinity, washes out the antibody that the affinity is low in the antibody. The antibody in the set is ranked according to the sort ascending of affinity. Through the specified evolution generation, the optimal antibody can be distilled. Therefore the optimal solution is obtained.

4.1 Basic Conception

In order to solve the problems, it is necessary to find the optimal solution of the problem needed by solving. The optimal solution is abstracted as the antigen, and the feasible solution of the problem is abstracted as an antibody that represents a candidate solution of the problem. For convenience to discuss the problem, here some definitions would be given as the following.

Definition 1. The antigen is specified as the optimized objective function.

Definition 2. The antibody is specified as the candidate solution of objective function. In the real number encoding, usually the antibody is a multi-dimension vector, $X = \upsilon X1, X2, ...XN)$, each antibody is represented by a point in n-dimension space.

Definition 3. The affinity of antibody-antigen is the value after computing antibody to replace antigen (optimized objective function).

4.2 Model of Clone Selection

In the human body immune system, when the antigen is in inbreak for human body, the immune system of human body would bring lots of antibody to match the antigen. Meanwhile, the antibody density, which affinity with antigen is large, would get higher, and it is propitious to eliminate the antigen. When the antigen dies out, this kind of antibody reproduction would be restrained. And at the same time, the antibody density would be reduced so as to make the immune system keep the balance all the time. In the antibody set, the superiority antibody, which is large in affinity with antigen, is activated. In order to eliminate antigen carrying through large number of clone, the encoding of clone selection process is designed as the following.

```
Procedure CloneSelect ( )
Begin
Assume antigen ag; /* s.t. min(J) */
Antibody set of random initialization;
While (evolution generation < m) / *m is the evolution generation * /
Begin
Computing f affinity (ag, ab) for each antibody until condition end,
Carrying through sort ascending array according to the affinity value
Selecting the front θ antibody brings new antibody ab new according to
the f num (ab(i))
Carrying through mutation for new brought antibody ab new
Joining N new antibody of random bringing into ab new
While (ab new non-empty)
Begin
Selecting the least affinity cell from the antibody set;
If (the affinity of selected cell > f affinity ( ag, ab new )
Then make replacing using new antibody;
    End
  End
End
Output the optimal solution from antibody set;
End
```

To make the new producing cell (come from clone selection process [7]) join to antibody set, the antibody density would be increased. It shows that the amount of approximate solution is increasing. But if this kind of antibody is excessively centralized then it is very difficult to keep antibody diversity. And the antibody owned better evolution potential would be lost, therefore it is able to go into the local optimization [8].

To avoid getting in local optimization, it is restricted to the amount of cloned memory cell in the clone selection process, shown as in formula (3).

$$f_{num} = \sum_{i=1}^{q} \lceil \frac{\beta \cdot \theta}{i} \rceil \tag{3}$$

In which, f num is the total clone number, the ith item represents the clone number of ith cell, β is a parameter factor of pre-enactment, θ is the amount of superiority cell. It can know from formula (3), the large the cell affinity is, also the more the clone amount is, and contrarily it would be less. For instance, β= 2, θ= 100, because the affinity magnitude of cloned memory cell is to arrange according to the sequence, therefore the cloned amount of maximum antibody cell of the affinity is 200, the cloned number is 100, the rest may be deduced by analogy method. Meanwhile in order to prevent getting in local optimization, it introduces certain new antibody produced by random in each generation evolution process.

The antibody cell updates dynamically in the evolutionary process. Each antibody cell always selects optimal antibody from the current antibody and new cloned antibody. Accordingly the dynamic update of antibody cell set is realized, and the antibody scale keeps the steady-state balance.

4.3 Selection for Mutation

The objective of mutation is to make change for encoding of filial generation antibody so as to obtain the better solution than the father generation. Because the antibody adopts real number encoding, so the mode of Gaussian mutation is used in the algorithm. And also the mutation does not be acted on barbarism species. In order to centralize search around high affinity antibody, and to guarantee the antibody diversity, it is introduced to a sort of self-adaptive mutation, namely it acts on individual component for each mutation operator shown as in formula (4).

$$X_i = |x_i + N_m * N(0, 1) * x_i| \tag{4}$$

In which, N(0,1) is a random number subjected to the standard Gaussian distribution, $|\cdot|$ is to find absolute value because of control parameter being not able to be negative, Nm is the mutation rate corresponding to the antibody, it is determined by formula (5).

$$Nm_i = \rho \frac{f(x_i)}{max(f(x_i))} \tag{5}$$

Obviously the mutation rate of the antibody is inversely proportional to its affinity, the higher the affinity is, and the smaller the mutation rate is. The antibody adjusts adaptively the mutation step-length in terms of affinity magnitude for each iterative process. It makes search to be centralized around the high affinity antibody so as to enhance the convergent speed and also it keeps the species diversity. The is the mutation constant used as adjusting the mutation intensity, it is related to the search space size and species scale.

5 Simulation and Its Analysis

For the above algorithm, here we can take the parameter tuning of human simulated intelligent controller (HSIC) as an examples to validate its correctness. For validating the performance of algorithm proposed in this paper, the controlled object [6] is selected as high order process, its transfer function is expressed by formula (7).

$$G_1(s) = 1/(1+s)8 \tag{6}$$

By means of the proposed method as mentioned above, it can be used as the tuning of HSIC algorithm [9]. The control algorithm is expressed as the formula (7).

$$\mu_{u_n} = \begin{cases} sgn(e_n)U_{max} \quad (e,\dot{e})\epsilon\Phi_1 \\ K_p * e_n \quad (e,\dot{e})\epsilon\Phi_2 \\ K_p * \dot{e}_n + K_d * \dot{e}_n \quad (e,\dot{e})\epsilon\Phi_3 \cup \Phi_6 \\ -K_p^{'} * \dot{e}_n + K_{d'} * \dot{e}_n \quad (e,\dot{e})\epsilon\Phi_4 \\ K_p * \dot{e}_n + K_i * \int e \quad (e,\dot{e})\epsilon\Phi_5 \end{cases} \tag{7}$$

where:
u_n : The Nth output of the controller,
e_n : The Nth deviation,
$\dot{e}_n$: The Nth deviation change rate,
$K_p, K_{p'}$: Proportional coefficient,
$K_d, K_{d'}$: Differential coefficient,
K_i : Integral coefficient,
U_{max} : The maximum output of the controller,
e_1, e_2 : The threshold of the deviation,
$\dot{e}_1$: The threshold of the deviation change rate

From the formula (7), it can be seen that there are seven parameters needed to be tuned, they are respectively, K_p, K_p', K_d, K_d', K_i and e_1, e_2. It is very difficult to tune the seven parameters of HSIC. In this paper, it selects a high order process as controlled plant [10] its transfer function is given in formula (6).

Firstly, the parameter needed to be optimized should be encoded, in the paper, real value encoding is adopted. Encoding K_p, K_p', K_d, K_d', K_i and $e_1, e_2, \dot{e}_1$ as an antibody, the antibody evolution generation is 100, population size is 50 the initial range of parameter, K_p and K_p' are over interval [0, 30], K_d, K_d' and K are over interval [0, 5], $e_1, e_2, \dot{e}_1$ are over interval [0, 1]. The others are respectively $\omega_1 = 0.999$, $\omega_2 = 0.001$, $\omega_4 = 100$, $\omega_3 = 2.0$, $\theta = 20$, $\beta = 5$, To increase global search ability, the five antibodies are added in each generation.

The paper realized the simulation of optimization control in MATLAB by means of optimization strategy based on immune mechanism, and we compared it with Chien-Hrones-Reswick (CHR) , Refined Ziegler Nichols (RZN) and Genetic Programming with ZN(GP) to tune controllers their control parameters come from Ref. [6].

Table 2 Parameters and objective values obtained of G1(s)

Parameters and variables	IHSIC	GP	RZN	CHR
KP		0.68	0.35	1.48
Ti	*	4.67	4.53	9.06
Td		1.47	1.14	2.02
ess (%)	0.00	0.00	0.00	0.00
mp (%)	0.00	4.40	0.00	48.54
ts (%)	13.59	10.02	30.04	23.22

∗ 0.43, 27.56, 1.59, 11.27, 0.01, 0.27, 0.25, 0.50

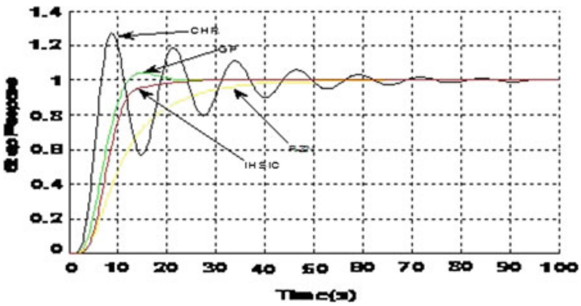

Fig. 2 Comparison curve for different algorithm

Table 2 is corresponding to the formula (7), for convenience contrast to the PID, the parameters in the Table 2 is still to use KP, Ti, Td to express, but the parameters of HSIC is expressed as "∗", the actual value is under Table 2, they are respectively the above seven parameters. In the Fig. 2 and Table 2, where ess represents steady state error, mp represents overshoot, and ts is the settling time. From Fig. 2, it can be concluded that IHSIC adjusting time is much better than CHR and RZN tuning PID control method, and the overshoot is obvious less than CHR. Compare with GP tuning PID, although the adjusting time is less than GP, but IHSIC has no overshoot.

6 Conclusion

Based on the clone selection mechanism of biology immune system, the paper proposed a sort of immune based optimization method for controller parameter tuning, and established the algorithm model that could solve the parameter tuning problems of control system. And in this paper, it is realized to the parameter tuning of PID and HSIC by the proposed immune model. The simulation experiment result shows that it owns the generality and effectiveness in method, and also it is suitable for parameter tuning of other algorithm such as optimal control etc.

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Preinvex Fuzzy-valued Function and Its Application in Fuzzy Optimization

Zeng-tai Gong, Yu-juan Bai and Wen-qing Pan

Abstract Based on the ordering of fuzzy numbers proposed by Goetschel and Voxman, in this paper, the representations and characterizations of semi-E-preinvex fuzzy-valued function are defined and obtained. As an application, the conditions of strictly local optimal solution and global optimal solution in the mathematical programming problem are discussed.

Keywords Fuzzy numbers · semi-E-preinvexity · fuzzy optimization

1 Introduction

The concept of fuzzy set was introduced by Zadeh in [11]. Since then, many applications of fuzzy set have been widely developed. Just as many systems with parameter uncertainty, the optimization theory with parameter uncertainty such as in objective function, constraints, or both of objective function and constraints, is often dealt. It is well known that the classical theory of convex analysis and mathematical programming are closely linked each other. Some authors have discussed the convexity, quasi-convexity and $B-$convex of fuzzy mappings [6, 8]. In 1994, Noor [5] introduced the concept of preinvex fuzzy-valued functions over the field of real numbers R, and obtained some properties of preinvex fuzzy-valued functions. After that, the properties of preinvex fuzzy-valued functions have been developed and generalized by many authors [7–10] and applied in fuzzy optimization problem [3]. The essence

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of preinvex fuzzy-valued functions is investigated and some judge theorems and a characterization of preinvex fuzzy-valued functions are obtained by using the upper (lower) semi-continuity [4]. In this paper, some representations and characterizations of semi-E-preinvex fuzzy-valued functions are obtained. As an application, the conditions of strictly local optimal solution and global optimal solution in the mathematical programming problem are discussed.

2 Preliminaries and Definitions

A fuzzy number is a mapping u: $R \to [0, 1]$, with the following properties:

1. u is normal, i.e., there exists $x_0 \in R$ with $u(x_0) = 1$;
2. u is convex fuzzy set;
3. u is semicontinuous on R and
4. $\overline{x \in R : u(x) > 0}$ is compact.

Let $\mathcal{F}$ be the set of all fuzzy numbers on R. For $u \in \mathcal{F}$, we write

$$[u]^\alpha = [u^-(\alpha), u^+(\alpha)],$$

then the following conditions are satisfied:

1. $u^-(\alpha)$ is abounded left continuous non-decreasing function on $(0, 1]$;
2. $u^+(\alpha)$ is abounded left continuous non-increasing function on $(0, 1]$;
3. $u^-(\alpha)$ and $u^+(\alpha)$ are right continuous at $\alpha = 0$ and left continuous at $\alpha = 1$.
4. $u^-(1) \leq u^+(1)$.

Conversely, if the pair of functions $u^-(\alpha)$ and $u^+(\alpha)$ satisfy conditions (1) – (4), then there exists a unique $u \in \mathcal{F}$ such that

$$[u]^\alpha = [u^-(\alpha), u^+(\alpha)]$$

for each $\alpha \in [0, 1]$ (see, e.g.[1])

For brevity, we write

$V = \{(u^-(\alpha), u^+(\alpha), \alpha) | 0 \leq \alpha \leq 1, u^- : I \to R, u^+ : I \to R$ are bounded functions$\}$; $\hat{V} = \{(u^-(\alpha), u^+(\alpha), \alpha) | 0 \leq \alpha \leq 1, u^-(\alpha), u^+(\alpha)$ are Lebesgue integrable$\}$; $\mathcal{F} = \{(u^-(\alpha), u^+(\alpha), \alpha) | 0 \leq \alpha \leq 1, u^-(\alpha)$ is left continuous non-decreasing function, $u^+(\alpha)$ is left continuous non-increasing function and they are right continuous at $\alpha = 0\}$.

The addition and scalar multiplication in V are defined as follows:

$(u^-(\alpha), u^+(\alpha), \alpha) + (v^-(\alpha), v^+(\alpha), \alpha) = (u^-(\alpha) + v^-(\alpha), u^+(\alpha) + v^+(\alpha), \alpha)$,
$k(u^-(\alpha), u^+(\alpha), \alpha) = (ku^-(\alpha), ku^+(\alpha), \alpha)$.

Let $x = (x_1, x_2, \cdots x_n) \in R^n$, $u = (u_1, u_2, \cdots u_n) \in \hat{V}^n$, then scalar product of x and u is defined by

$$\langle x, u \rangle = x_1 u_1 + x_2 u_2 + \cdots + x_n u_n.$$

Assume that $u, v \in \hat{V}$,

$$u = \{(u^-(\alpha), u^+(\alpha), \alpha) | 0 \le \alpha \le 1\},$$

$$v = \{(v^-(\alpha), v^+(\alpha), \alpha) \, \| 0 \le \alpha \le 1\}$$

are members of $\hat{V}$, then u precedes v ($u \le v$) if

$$\int_0^1 f(\alpha)(u^-(\alpha) + u^+(\alpha))d\alpha \le \int_0^1 f(\alpha)(v^-(\alpha) + v^+(\alpha))d\alpha.$$

For a fuzzy-valued function

$$F(x) = \{(F^-(\alpha, x), F^+(\alpha, x), \alpha) | 0 \le \alpha \le 1\},$$

we define

$$T_{F(x)} = \int_0^1 f(\alpha)[F^-(\alpha, x) + F^+(\alpha, x)]d\alpha,$$

where f is monotone non-decreasing function and $f(0) = 1$, $\int_0^1 f(\alpha)d\alpha = \frac{1}{2}$. f can be interpreted as weighting function, its nondecreasing ensure a closer level of nuclear cuts and determine the relationship between the greater role. Especially, if $f(\alpha) = \alpha$, then the ordering relation between two $u, v \in \hat{V}$ could be found in [2] which defined by Goetschel and Voxman.

Definition 2.1 [5] *Let $S \subset R^n$ be an open set, $F : S \to \mathcal{F}$ be a fuzzy-valued function. If $\frac{\partial}{\partial x_i} F^-(\alpha, x)$ and $\frac{\partial}{\partial x_i} F^+(\alpha, x)$ $(i = 1, 2, \ldots, n)$ are continuous, then $F(x)$ is said to be differentiable on S, and*

$$\nabla F(x) = \left[\frac{\partial}{\partial x_1} F(x), \frac{\partial}{\partial x_2} F(x), \ldots, \frac{\partial}{\partial x_n} F(x) \right]$$

is called the gradient of fuzzy-valued function $F(x)$.

Let $y \in (S \subset R^n)$, we say S is invex at y with respect to $\eta : S \times S \to R^n$, if for each $x \in S$, $\lambda \in [0, 1]$, $y + \lambda\eta(x, y) \in S$.

S is said to be an invex set with respect to η if S is invex at each $y \in S$.

In particular, when $\eta(x, y) = x - y$, invex set degrades a general convex set.

$\eta : S \times S \to R^n$ is called a skew mapping if

$$\eta(x, y) + \eta(y, x) = 0, x, y \in S, \ \lambda \in [0, 1].$$

Definition 2.2 *A fuzzy-valued function $F : S \to \mathcal{F}$ is said to be preinvex on invex set S with respect to $\eta : S \times S \to R^n$, if for any $x, y \in K$, $\lambda \in [0, 1]$,*

$$T_{F(y+\lambda\eta(x,y))} \leq \lambda T_{F(x)} + (1-\lambda)T_{F(y)}.$$

Definition 2.3 *A set $S(\subset R^n)$ is said to be E-invex set with respect to $\eta : S\times S \to R^n$ on S, if there is a mapping $E : R^n \to R^n$ such that*

$$E(y) + \lambda\eta(E(x), E(y)) \in S,$$

for $0 \leq \lambda \leq 1$.

Definition 2.4 *A fuzzy-valued function $F : S \to \mathcal{F}$ is said to be semi-E-preinvex respect to $\eta : S \times S \to R^n$ on S, if there is a mapping $E : R^n \to R^n$ such that*

$$E(y) + \lambda\eta(E(x), E(y)) \in S$$

and

$$T_{F(E(y)+\lambda\eta(E(x),E(y)))} \leq \lambda T_{F(x)} + (1-\lambda)T_{F(y)}$$

for each $x, y \in S$ and $0 \leq \lambda \leq 1$.

3 The Judgement and Characterization of Semi-E-preinvex Fuzzy-valued Functions

Theorem 3.1 *Let $F : S \to \mathcal{F}$ be a semi-E-preinvex fuzzy-valued function on E-invex set S, then $T_{F(E(y))} \leq T_{F(y)}$ for each $x \in S$.*

Proof. Let $F : S \to \mathcal{F}$ be a semi-E-preinvex fuzzy-valued function on E-invex set S and $\lambda \in [0, 1]$. Then

$$T_{F(E(y)+\lambda\eta(E(x),E(y)))} \leq \lambda T_{F(x)} + (1-\lambda)T_{F(y)}.$$

Thus for $\lambda = 0$, then $T_{F(E(y))} \leq T_{F(y)}$ for each $x \in S$.

Theorem 3.2 *Let $F : S \to \mathcal{F}$ be a semi-E-preinvex fuzzy-valued function on S iff*

$$T_{F(E(y)+\lambda\eta(E(x),E(y)))} \leq \lambda T_u + (1-\lambda)T_v$$

for $u, v \in \mathcal{F}$ satisfying $T_{F(x)} \leq T_u$, $T_{F(y)} \leq T_v$, and $x, y \in S$, where $0 \leq \lambda \leq 1$.

Proof. Note that $T_{F(x)} < T_u$, $T_{F(y)} < T_v$, and F is semi-E-preinvex on S, then

$$T_{F(E(y)+\lambda\eta(E(x),E(y)))} \leq \lambda T_{F(x)} + (1-\lambda)T_{F(y)} < \lambda T_u + (1-\lambda)T_v.$$

Conversely, write

$$F(x) = (F^-(\alpha, x), F^+(\alpha, x), \alpha),\ F(y) = (F^-(\alpha, y), F^+(\alpha, y), \alpha)$$

for $x, y \in S, \lambda \in (0, 1)$. $\forall \varepsilon > 0$, let

$$u = \{(F^-(\alpha, x) + \frac{\varepsilon}{2}, F^+(\alpha, x) + \frac{\varepsilon}{2}, \alpha) |\ 0 \le \alpha \le 1\},$$

$$v = \{(F^-(\alpha, y) + \frac{\varepsilon}{2}, F^+(\alpha, y) + \frac{\varepsilon}{2}, \alpha) |\ 0 \le \alpha \le 1\},$$

then

$$T_{F(x)} = \int_0^1 \alpha[F^-(\alpha, x) + F^+(\alpha, x)]d\alpha < \int_0^1 \alpha[F^-(\alpha, x) + F^+(\alpha, x) + \varepsilon]d\alpha = T_u,$$

$$T_{F(y)} = \int_0^1 \alpha[F^-(\alpha, y) + F^+(\alpha, y)]d\alpha < \int_0^1 \alpha[F^-(\alpha, y) + F^+(\alpha, y) + \varepsilon]d\alpha = T_v.$$

That is,

$$T_{F(E(y)+\lambda\eta(E(x),E(y)))}$$

$$< \lambda \int_0^1 \alpha[F^-(\alpha, x) + F^+(\alpha, x) + \varepsilon]d\alpha + (1-\lambda) \int_0^1 \alpha[F^-(\alpha, y) + F^+(\alpha, y) + \varepsilon]d\alpha.$$

It follows that

$$T_{F(E(y)+\lambda\eta(E(x),E(y)))} \le \lambda T_{F(x)} + (1 - \lambda)T_{F(y)}$$

when $\varepsilon \to 0$.

Theorem 3.3 *Let $F : S \to \mathcal{F}$ be a semi-E-preinvex fuzzy-valued function on E-invex set S. Then*

$$K_u(F) = \{x | x \in S,\ T_{F(x)} \le T_u\} \quad \forall u \in \mathcal{F}$$

is E-invex set.

Proof. For any $x, y \in K_u(F)$, we have $T_{F(x)} \le T_u$, $T_{F(y)} \le T_u$. Since S is E-invex set, i.e.

$$E(y) + \lambda\eta(E(x), E(y)) \in S,$$

and $F : S \to \mathcal{F}$ is semi-E-preinvex on E-invex set S, i.e.

$$T_{F(E(y)+\lambda\eta(E(x),E(y)))} \le \lambda T_{F(x)} + (1 - \lambda)T_{F(y)} \le T_u,$$

we have

$$E(y) + \lambda\eta(E(x), E(y)) \in K_u(F).$$

Hence $K_u(F)$ is a E-invex set.

Given a mapping $E : R^n \to R^n$. Let the mapping

$$E \times I : R^n \times \mathcal{F} \to R^n \times \mathcal{F}$$

to be

$$E \times I(x, u) = (E(x), u), \quad \forall (x, u) \in R^n \times \mathcal{F}.$$

Definition 3.1 *Let $S \subset R^n \times \mathcal{F}$. S ($\subset R^n \times \mathcal{F}$) is said to be a $E \times I$-invex set if there exists $E : R^n \to R^n$ such that*

$$\begin{aligned} &E \times I(y, v) + \lambda\eta[E \times I(x, u), E \times I(y, v)] \\ &\quad = [E(y) + \lambda\eta(E(x), E(y)), \lambda u + (1 - \lambda)v] \in S, \end{aligned}$$

for $(x, u), (y, v) \in S$ ($x, y \in R^n,\ u, v \in \mathcal{F}$) and $\lambda \in [0, 1]$.

Theorem 3.4 *Let $\{S_i\}_{i \in J}$ ($S_i \subset R^n \times \mathcal{F}$) be $E \times I$-invex set. Then $\bigcap_{i \in J} S_i (\subset R^n \times \mathcal{F})$ is a $E \times I$-invex set.*

Proof. Let $(x, u), (y, v) \in \bigcap_{i \in J} S_i$, $\lambda \in [0, 1]$, then $\forall i \in J$, we have (x, u), $(y, v) \in S_i$. Since each $S_i (\subset R^n \times \mathcal{F})$ is $E \times I$-invex, i.e. there exists a mapping $E : R^n \to R^n$ such that for $(x, u), (y, v) \in S_i$ and $\lambda \in [0, 1]$, we have

$$\begin{aligned} &E \times I(y, v) + \lambda\eta[E \times I(x, u), E \times I(y, v)] \\ &\quad = [E(y) + \lambda\eta(E(x), E(y)), \lambda u + (1 - \lambda)v] \in S_i \end{aligned}$$

for $i \in J$. It follows that

$$[E(y) + \lambda\eta(E(x), E(y)), \lambda u + (1 - \lambda)v] \in \bigcap_{i \in J} S_i.$$

That is, $\bigcap_{i \in J} S_i (\subset R^n \times \mathcal{F})$ isa $E \times I$-invex set.

Theorem 3.5 *Let S be a $E-$invex set. Then F is a semi-E-preinvex fuzzy-valued function on S iff*

$$\{(x, u) | x \in S\ , u \in \mathcal{F},\ T_{F(x)} < T_u\}$$

is a $E \times I-$ invex set.

Proof. Let $S(F) = \{(x, u) | x \in S\ , u \in \mathcal{F},\ T_{F(x)} < T_u\}$. Since

$$\begin{aligned} &E \times I(y, v) + \lambda\eta[E \times I(x, u), E \times I(y, v)] \\ &\quad = [E(y) + \lambda\eta(E(x), E(y)), \lambda u + (1 - \lambda)v] \in S(F), \end{aligned}$$

i.e. for any $x, y \in S$, $\lambda \in [0, 1]$ and $u, v \in \mathcal{F}$ satisfying $T_{F(x)} < T_u,\ T_{F(y)} < T_v$, thus

$$T_{F(E(y)+\lambda\eta(E(x),E(y)))} < \lambda T_u + (1-\lambda)T_v.$$

According to Theorem 3.2, F is semi-E-preinvex on S iff $S(F)$ is a $E \times I-$ invex set.

We define an epigraph of F as follows:

$$epi(F) = \{(x,u)|x \in S\ , u \in \mathcal{F},\ T_{F(x)} \leq T_u\}.$$

Theorem 3.6 *Let S be an $E-$invex set. Then $F : S \to \mathcal{F}$ is a semi-E-preinvex fuzzy-valued function on S iff*

$$epi(F) = \{(x,u)|x \in S\ , u \in \mathcal{F},\ T_{F(x)} \leq T_u\}$$

is an $E \times I-$ invex set.

Proof. Let F be a semi-E-preinvex fuzzy-valued function on S. Then for any (x,u), $(y,v) \in epi(F)$ and $\lambda \in [0,1]$, we have

$$E(y) + \lambda\eta(E(x), E(y)) \in S$$

and

$$T_{F(E(y)+\lambda\eta(E(x),E(y)))} \leq \lambda T_{F(x)} + (1-\lambda)T_{F(y)} \leq \lambda T_u + (1-\lambda)T_v.$$

Hence

$$\begin{aligned} &E \times I(y,v) + \lambda\eta[E \times I(x,u), E \times I(y,v)] \\ &\quad = [E(y) + \lambda\eta(E(x), E(y)), \lambda u + (1-\lambda)v] \in epi(F). \end{aligned}$$

i.e. $epi(F)$ is an $E \times I-$ invex set.

Conversely, $\forall (x,y) \in S$, $\lambda \in [0,1]$, we have $(x, F(x)) \in epi(F)$, $(y, F(y)) \in epi(F)$. Since $epi(F)$ is an $E \times I-$ invex set, we have

$$\begin{aligned} &E \times I(y, F(y)) + \lambda\eta[E \times I(x, F(x)), E \times I(y, F(y))] \\ &\quad = [E(y) + \lambda\eta(E(x), E(y)), \lambda F(x) + (1-\lambda)F(y)] \in epi(F). \end{aligned}$$

Hence

$$T_{F(E(y)+\lambda\eta(E(x),E(y)))} \leq \lambda T_{F(x)} + (1-\lambda)T_{F(y)}.$$

Therefore, F is semi-E-preinvex on S.

Theorem 3.7 *Let $\{F_i|i \in J\}$ be a collection of semi-E-preinvex function on S, if for any $x \in S$, $\sup\{F_i(x)|i \in J\}$ exists, then $F(x) = \sup\{F_i(x)|i \in J\}$ is semi-E-preinvex on S.*

Proof. $\forall i \in J$, $\{F_i\}$ is semi-E-preinvex on S, then

$$epi(F_i) = \{(x, u)|x \in S\ , u \in \mathcal{F},\ T_{F_i(x)} \leq T_u\}$$

is an $E \times I-$ invex set, Furthermore,

$$\bigcap_{i \in J} epi(F_i) = \{(x, u)|x \in S\ , u \in \mathcal{F},\ T_{F_i(x)} \leq T_u,\ i \in J\}$$

is an $E \times I-$ invex set. Note that

$$\begin{aligned}
&(x, u) \in \bigcap_{i \in J} epi(F_i) \\
&\Leftrightarrow x \in S\ , u \in \mathcal{F},\ T_{F_i(x)} \leq T_u, i \in J \\
&\Leftrightarrow x \in S\ , u \in \mathcal{F},\ T_{F(x)} \leq T_u \\
&\Leftrightarrow (x, u) \in epi(F).
\end{aligned}$$

Hence $\bigcap_{i \in J} epi(F_i) = epi(F)$ is an $E \times I-$ invex set. According Theorem 3.6, F is semi-E-preinvex on S.

Theorem 3.8 *Let $F_i : S \to \mathcal{F}$ $(i = 1, 2, \cdots, k)$ be a semi-E-preinvex fuzzy-valued function on S with a mapping $E : R^n \to R^n$. Then*

$$h(x) = \Sigma_{i=1}^{k} a_i F_i(x)\ (a_i \geq 0,\ i = 1, 2, \cdots, k)$$

is semi-E-preinvex on S.

Proof. Since $F_i : S \to \mathcal{F}$ $(i = 1, 2, \cdots, k)$ is semi-E-preinvex on S with a mapping $E : R^n \to R^n$, i.e. for any $x, y \in S$ and $\lambda \in [0, 1]$, then

$$T_{F_i(E(y)+\lambda\eta(E(x),E(y)))} \leq \lambda T_{F_i(x)} + (1 - \lambda)T_{F_i(y)},\ \ i = 1, 2, \cdots, k.$$

Hence

$$T_{\Sigma_{i=1}^{k} F_i(E(y)+\lambda\eta(E(x),E(y)))} \leq \lambda T_{\Sigma_{i=1}^{k} F_i(x)} + (1 - \lambda)T_{\Sigma_{i=1}^{k} F_i(y)}.$$

Therefore

$$T_{h(E(y)+\lambda\eta(E(x),E(y)))} \leq \lambda T_{h(x)} + (1 - \lambda)T_{h(y)}.$$

i.e. h is semi-E-preinvex on S.

Theorem 3.9 *Let F be semi-E-preinvex on S. Then the following states are true:*

1. If $\phi : \mathcal{F} \to \mathcal{F}$ *is a nondecreasing convex function, then the composite function* $\phi \circ F : S \to \mathcal{F}$ *is semi-E-preinvex on* S;
2. If $\phi : \mathcal{F} \to \mathcal{F}$ *is positively homogeneous nondecreasing additive function, then the composite function* $\phi \circ F : S \to \mathcal{F}$ *is semi-E-preinvex on* S.

Proof. For any $x, y \in S,\ \lambda \in [0, 1]$, we have $E(y) + \lambda\eta(E(x), E(y)) \in S$ and

$$T_{F(E(y)+\lambda\eta(E(x),E(y)))} \leq \lambda T_{F(x)} + (1 - \lambda)T_{F(y)},$$

1. When $\phi : \mathcal{F} \to \mathcal{F}$ is a nondecreasing convex function,

$$\begin{aligned}
&T_{\phi\circ F(E(y)+\lambda\eta(E(x),E(y)))}\\
&= T_{\phi(F(E(y)+\lambda\eta(E(x),E(y))))}\\
&\leq T_{\phi(\lambda F(x)+(1-\lambda)F(y))}\\
&\leq T_{\lambda\phi F(x)+(1-\lambda)\phi F(y)}\\
&= T_{\lambda\phi\circ F(x)+(1-\lambda)\phi\circ F(y)}.
\end{aligned}$$

   i.e. $\phi \circ F : S \to \mathcal{F}$ is semi-E-preinvex on S.
2. When $\phi : \mathcal{F} \to \mathcal{F}$ be positively homogeneous nondecreasing additive function,

$$\begin{aligned}
&T_{\phi\circ F(E(y)+\lambda\eta(E(x),E(y)))}\\
&= T_{\phi(F(E(y)+\lambda\eta(E(x),E(y))))}\\
&\leq T_{\phi(\lambda F(x)+(1-\lambda)F(y))}\\
&\leq T_{\phi(\lambda F(x))+\phi((1-\lambda)F(y))}\\
&\leq T_{\lambda\phi F(x)+(1-\lambda)\phi F(y)}\\
&= T_{\lambda\phi\circ F(x)+(1-\lambda)\phi\circ F(y)}.
\end{aligned}$$

   i.e. $\phi \circ F : S \to \mathcal{F}$ is semi-E-preinvex on S.

4 The Optimization of Preinvex Fuzzy-valued Function

Let $F : S \to \mathcal{F}$ be a fuzzy-valued function on S. We consider the following fuzzy optimization problem.

$$(P)\ \ \min\ F(x),\ s.t.\ x \in S = \{x \in R^n | G_i(x) \leq \tilde{0}, i = 1, 2, \ldots, m\},$$

where $F : R^n \to \mathcal{F}$ and $G_i : R^n \to \mathcal{F}$ are semi-E-preinvex on S.

Theorem 4.1 *Let $F : R^n \to \mathcal{F}$ and $G_i : R^n \to \mathcal{F}$ be semi-E-preinvex fuzzy-valued functions on R^n. Then S is $E-$invex.*

Proof. Let $G_i : R^n \to \mathcal{F}$ $(i = 1, 2, \ldots, m)$ be semi-E-preinvex fuzzy-valued function on R^n,

$$S_i = \{x \in R^n | T_{G_i(x)} \leq 0\}\ (i = 1, 2, \ldots, m).$$

Then

$$T_{G_i(E(y)+\lambda\eta(E(x),E(y)))} \leq \lambda T_{G_i(x)} + (1-\lambda)T_{G_i(y)} \in S_i.$$

Hence

$$S = \bigcap_{i=1}^{m} S_i = \{x \in R^n | T_{G_i(x)} \leq 0, i = 1, 2, \dots, m\}$$

is $E-$invex.

Similarly to the proof of Theorem 4.1, we have the following theorem.

Theorem 4.2 *Let $F : R^n \to \mathcal{F}$ and $G_i : R^n \to \mathcal{F}$ be semi-E-preinvex fuzzy-valued function on R^n. Then $F : R^n \to \mathcal{F}$ is semi-E-preinvex on S.*

Theorem 4.3 *Let $F : R^n \to \mathcal{F}$ be semi-E-preinvex on S and $\bar{x}$ a solution of the following problem:*

$$(P_E) \quad \min(F \circ E)(x), \qquad x \in S.$$

Then $E(\bar{x})$ is a solution of the problem P.

Proof. Let $E(\bar{x})$ be a nonsolution of the problem P. Then there exists $y \in S$ such that

$$T_{F(y)} < T_{F(E(\bar{x}))}.$$

Then according 4.1,we have $T_{F(E(y))} \leq T_{F(y)}$. Hence

$$T_{F(E(y))} < T_{F(E(\bar{x}))},$$

which contradicts the optimality of $\bar{x}$ for the problem (P_E). Therefore $E(\bar{x})$ be a solution of problem P.

Theorem 4.4 *Let $F : R^n \to \mathcal{F}$ be semi-E-preinvex fuzzy-valued function on S and $\bar{x} = E(\bar{x}) \in S$ a local solution of the problem of (P). Then $\bar{x}$ be a global solution of problem P.*

Proof. Let $\bar{x} = E(\bar{x}) \in S$ be a local solution of the problem of (P). Then there exists $\delta > 0$, such that $\forall x \in U(\bar{x}, \delta) \bigcap S$, we have

$$T_{F(\bar{x})} < T_{F(x)}.$$

Suppose $\bar{x}$ is a nonsolution of problem of (P), then there exists $y \in S$ such that

$$T_{F(y)} < T_{F(\bar{x})} = T_{F(E(\bar{x}))}.$$

$\forall \lambda \in (0, 1)$, we have

$$\lambda T_{F(y)} + (1-\lambda)T_{F(\bar{x})} < T_{F(\bar{x})}.$$

Since F is semi-E-preinvex on S and $\bar{x} = E(\bar{x}) \in S$, we have

$$T_{F(E(\bar{x})+\lambda\eta(E(y),E(\bar{x})))} < \lambda T_{F(y)} + (1-\lambda)T_{F(\bar{x})}.$$

Hence

$$T_{F(E(\bar{x})+\lambda\eta(E(y),E(\bar{x})))} < T_{F(\bar{x})}.$$

Since λ may be arbitrarily small, we have

$$E(\bar{x}) + \lambda\eta(E(y), E(\bar{x})) \in U(\bar{x}, \delta) \bigcap S.$$

which contradicts (1), therefore $\bar{x}$ is a global solution of problem P.

Theorem 4.5 *Let $F : R^n \to \mathcal{F}$ be semi-E-preinvex fuzzy-valued function on S and $\bar{x} \in S$ satisfy*

$$T_{F(E(\bar{x}))} = T_{\min_{x\in S} F(E(x))},$$

if $T_u = T_{\min_{x\in S} F(E(x))}$. Then $\Omega = \{x \in S | T_{F(x)} = T_u\}$ is $E-$invex.

Proof. $\forall x, y \in \Omega$, $0 \leq \lambda \leq 1$, then

$$x, y \in S, \quad T_{F(x)} = T_u, \quad T_{F(y)} = T_u.$$

Since $F : R^n \to \mathcal{F}$ is semi-E-preinvex on S, we have

$$E(y) + \lambda\eta(E(x), E(y)) \in S,$$

and

$$T_u \leq T_{F(E(y)+\lambda\eta(E(x),E(y)))} \leq \lambda T_{F(x)} + (1-\lambda)T_{F(y)} = T_u.$$

Hence

$$T_{F(E(y)+\lambda\eta(E(x),E(y)))} = T_u.$$

i.e.

$$E(y) + \lambda\eta(E(x), E(y)) \in \Omega,$$

therefore Ω is $E-$invex.

5 Conclusions

In the real world there are many linear programming problems where all decision parameters are fuzzy numbers. Many authors have considered various types of fuzzy linear programming problems and proposed several approaches for solving these problems. One of the approaches for solving fuzzy linear programming problems is based on the concept of comparison of fuzzy numbers by use of ranking functions. In this paper, we used the ordering of fuzzy numbers proposed by Goetschel and Voxman, obtained representations and characterizations of semi-E-preinvex fuzzy-valued function. As an application, the conditions of strictly local optimal solution and global optimal solution in the mathematical programming problem have been discussed.

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