Perspektiven der Mathematikdidaktik
Gabriele Kaiser · Rita Borromeo Ferri · Werner Blum *Hrsg.*

RESEARCH

Xinrong Yang

Conception and Characteristics of Expert Mathematics Teachers in China

Springer Spektrum

Perspektiven der Mathematikdidaktik

Herausgegeben von
G. Kaiser, Hamburg, Deutschland
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Conception and Characteristics of Expert Mathematics Teachers in China

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Dissertation University of Hong Kong, 2010

ISBN 978-3-658-03096-4 ISBN 978-3-658-03097-1 (eBook)
DOI 10.1007/978-3-658-03097-1

The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at http://dnb.d-nb.de.

Library of Congress Control Number: 2013944192

Springer Spektrum

Printed on acid-free paper

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Springer DE is part of Springer Science+Business Media.
www.springer-spektrum.de

Foreword

The superior performance of East Asian students in recent international studies of mathematics achievement has attracted the attention of educators and policy makers worldwide. One interesting phenomenon that has been observed is that these high performing countries share a similar culture, sometimes named the Confucian Heritage Culture or CHC. Because of this phenomenon, educators and researchers have been interested in gaining a better understanding of mathematics education in China, and substantial research has been conducted on various aspects of mathematics education in China, ranging from studies on the educational policies to the official curriculum and to classroom teaching. However, very few studies have focused on one of the most fundamental issues in mathematics education — the quality of mathematics teachers. In this regard, the exploratory study conducted by Dr. Xinrong Yang based on his PhD work and as reported in this book provides important information and insight for understanding this research gap.

As Xinrong's PhD supervisor, I am very glad to witness his book seeing the light of day. I can still remember seven years ago when Xinrong started his PhD study with me, he told me that he was interested in exploring how expert mathematics teachers in China develop their expertise. However, to achieve this goal, one fundamental issue to figure out is how an expert mathematics teacher is defined, as there is no clear consensus in the literature on the definition of an expert teacher. Subsequently, Xinrong modified his research focus to exploring how an expert mathematics teacher is conceptualized by educators in China and the characteristics that expert mathematics teachers in China share. I am glad that the final thesis is a very fine piece of scholarly work, and I am sure the work will make a valuable contribution to the literature in the field of teacher education and development.

As Xinrong reviewed and argued in his work, teacher expertise is a culture-bound notion. China, as a country with a rich culture through more than five thousand years of history, has a lot of unique characteristics regarding education in general and mathematics teaching in particular. The traditional Confucian culture, or CHC, is still asserting significant influence on education in China and in many East Asian countries today. However, as widely reported in the literature, the superior mathe-

matics performance of students in China and the rest of East Asia has been achieved despite rather unfavorable conditions such as large class size, and teacher-centered and examination-driven teaching. It is intriguing to learn how teachers develop their expertise and how the notion of expert teacher is conceived in such an unfavorable environment, as it can be expected that the conception and development of teacher expertise in this special context would be very different from other contexts. An appreciation of how teacher expertise is conceived in such a social and cultural context may provide the key for understanding other aspects of mathematics education in China, and may throw light on how teacher expertise and conception of expertise are influenced by the social and cultural context more generally.

Xinrong´s own learning experience as well as his experience in pre-service teacher education in China had enabled him to conduct his study with much insight. From his rich knowledge of the relevant literature, he adopted a sociocultural theory and a prototypical view of conception in this study of teacher expertise. He found that some of the roles expected to be played by expert mathematics teachers in China, such as being at the same time a researcher, a mentor, and an expert in examination, are quite different from the roles expected of an expert teacher in the Western culture. In addition, some characteristics of an expert teacher identified in his study are also different from the features reported in previous studies. Examples include the expert mathematics teachers´ beliefs about mathematics and its learning and teaching, and their ability to balance direct teaching and exploratory teaching.

Findings such as these should be of interest to those who are interested in mathematics education and teacher education in China, as well as those who are interested in the field of teacher expertise. Readers will no doubt gain other insights from this resourceful and inspiring book, and I am sure this book will be making an impact in the field in the years to come.

Frederick K. S. Leung

The University of Hong Kong

Abstract

This study explores: 1) how mathematics educators in mainland China conceptualize expert mathematics teachers; 2) characteristics of expert mathematics teachers; and 3) how the Chinese social and cultural context influences both. Taking a sociocultural perspective and adopting a prototype view of teaching expertise as its theoretical foundation, this study examines, through semi-structured interviews, the conception of expert mathematics teachers from the perspectives of eleven mathematics teachers, six (vice) school principals, two mathematics teacher educators, and two mathematics teaching research officers. Based on the 21 interviewees' recommendations, three expert mathematics teachers' beliefs, knowledge and teaching practices were investigated further for common characteristics. Five to six consecutive lessons in a particular class were observed and videotaped, with each of the teachers being interviewed before and after every lesson.

The constant comparative method (Glaser & Strauss, 1968) was adopted for data analysis. Characteristics mentioned by more than 50% of the 21 mathematics educators were considered as components of the conception of expert mathematics teachers, and features found in at least two of the three teachers were treated as prototypical features of expert mathematics teachers in mainland China.

It was found that expert mathematics teachers were conceptualized as teachers playing multiple roles, including demonstrating expertise in teaching, conducting research and publishing papers, and mentoring teachers. They should not only be knowledgeable in mathematics, theory, characteristics of learners, curriculum, and many other fields, but also be exemplary models for students and colleagues. Most of the characteristics described by the 21 interviewees were identified in the three expert mathematics teachers, except for some discrepancies in opinions about knowledge related to advanced mathematics and research ability. The three expert mathematics teachers were found to hold contemporary-constructivist oriented beliefs, and to possess a wide and profound knowledge base. They could teach with flexibility, balance, and coherence. They could promote students' higher order thinking and their teaching practice was consistent with the beliefs they held. They could systematically reflect on their teaching and propose modifications and improvements.

Results indicate that the concept of expert mathematics teachers is culturally bounded and that their teaching is influenced by the social and cultural context; however, they also demonstrate the ability to work against social and cultural constraints. This study's findings contribute to: 1) understanding of the conception of expert teachers in a particular subject and within a specific sociocultural context; 2) how a specific social and cultural contexts influence expert mathematics teachers' beliefs, knowledge, and practice; 3) a new perspective on mathematics education in China; 4) a new perspective on differences between the teaching of novice and expert teachers; and 5) curriculum development in pre-service and in-service teacher education. Further research is needed to explore the concept and characteristics of expert mathematics teachers at other grade levels and in other social and cultural contexts, to provide a deeper and fuller understanding of expert mathematics teachers.

Acknowledgements

This study would not have been possible without the guidance of my supervisor and panel members, help from my friends, and support from my parents.

I would like to express my deepest gratitude to my PhD supervisor, Professor Frederick K. S. Leung, for his excellent guidance, caring and patience, and for providing me with an excellent atmosphere in which to do research. During my four years of PhD study at the University of Hong Kong, he has continually inspired me, encouraged me to overcome difficulties, and offered insightful feedback on my research. Without his continuous guidance and support, this study would not have been possible.

I would also like to thank Dr. Allen Leung, Dr. Matthew Clarke, and Dr. Ida Mok, who kindly offered many suggestions on the theoretical foundation and research design of this study. Thanks also go to Professor Gabriele Kaiser for her enthusiastic encouragement, research guidance and deep interests in this study, and to Dr. Zhu Yan, who read most of the chapters and offered many useful suggestions. I am indebted, also, to many other scholars, including Professor Colette Laborde, Professor Celia Hoyles, Dr. Paola Valero and Dr. Bill Atweh, for their suggestions.

Many thanks go to Mr. Zhang Xiaobin, Mr. Zhu Furong, Professor Song Naiqing, Dr. Li Zhongru, Dr. Yu Bo, Dr. Huang Yanping, Dr. Kang Shigang, Dr. Yang Yuhui for their gracious assistance during my fieldwork in Chongqing. I would also like to express my deep appreciation to participants in the study, in particular Ms. Qian, Ms. Sun, and Mr. Zhao (all pseudonyms), who invested a great deal of time and effort in this study, and without whose cooperation it would not have been possible.

I am indebted to many officemates and friends who helped me with this study. Dr. Mtahabwa Lyabwene helped at the early stage of this study and offered many useful suggestions. Dr. Chan Yip Cheung and Dr. Gao Fang helped me to shape my ideas at the beginning and provided many suggestions on data analysis and reporting. Ms. Ma Jingjing, Ms. He Peichang, Ms. Gao Manman and Mr. Xie Ailei, also offered suggestions,

helped me to solve many language problems and polished my writing. I would also like to thank Dr. Gloria Stillman, who helped to polish my writing and made many comments on my revisions. My thanks also go to Ms. Liang Xiaohua for her enlightening discussions, and to Rachel, Chen Qian, Ding Lin, J.S., Doris, Connie, Samuel, Kitty and many other friends in the Faculty of Education and the Research Office for their unselfish help.

My thanks to my roommates, Denis Yueng, Zhang Jie and Jiang Wei, and to many other brothers from the fifth floor in Morrison Hall; my time with them should forever be a treasured memory. I would also like to thank Morrison Hall, which gave me a home for four years and alleviated many troubles during my studies.

Last but by no means least, I am heartily thankful to my family for their unswerving support. In particular, I would like to thank my parents for taking care of themselves so well, so that I could focus on my study.

Xinrong YANG

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Contents

Chapter Three

Chapter Four

List of Figures

List of Tables

Chapter One

Introduction

1.1 Background to the Problem

Since the 1980s, Chinese students, including those from mainland China, Hong Kong, and Taiwan, have consistently outperformed their Western counterparts in large-scale international studies in mathematics, such as IAEP, TIMSS, and PISA (Fan & Zhu, 2004; OECD, 2010). Students from mainland China once ranked the first in IAEP2 (Fan & Zhu, 2004) and in PISA 2009 (OECD, 2010), and recorded excellent performance in some small-scale comparison studies in mathematics achievement as well (*e.g.*, Lee, 1998; Stevenson *et al.*, 1993). In addition, students from mainland China have been champion in the International Mathematical Olympiads (IMO) many times (IMO, 2013). While these achievements are impressive, some Western researchers have found that the Chinese learning environment, with its large class size, expository teaching methods and focus on preparation for external examinations, does not appear to be conducive to effective learning (Biggs, 1996).

Chinese students' outstanding mathematics performance, despite their unfavourable educational environment, has been identified as the so-called "the Chinese Learner Paradox" (Biggs & Watkins, 1996; Marton *et al.*, 1993), and the paradox has drawn the attention of numerous researchers, including many from the West who were disappointed with their own students' mathematics achievement (Stevenson & Stigler, 1992). The researchers explored the paradox from various perspectives and hypothesized that the differences in number systems (Fuson & Kwon, 1991; Miller *et al.*, 1995), cultural contexts (*e.g.*, parental expectations and beliefs in ability), school organizations, and mathematics curricula might contribute to Chinese students' excellent achievement (Lee, 1998; Stevenson *et al.*, 1990; Stigler & Perry, 1988).

An additional important factor attributed to Chinese students' outstanding mathematics achievement might be "schooling, more specifically, the educational practices of teaching—learning mathematics at school" (Hatano & Inagaki, 1998, p. 82). This suggests that teacher quality may play a significant role in student learning. In fact, Moir *et al.* (2009) suggested teachers' quality is the most important school-related

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DOI 10.1007/978-3-658-03097-1_1, © Springer Fachmedien Wiesbaden 2014

factor in student learning outcomes, and "dwarfs every other school-related variable … including class size, school size, and even the heterogeneity of prior achievement within a classroom" (p. 11). Many other researchers (*e.g.*, An, 2004; Blömeke & Kaiser, 2012; Even *et al.*, 2003; Hargeaves, 1994; Leung & Park, 2002; Ma, 1999; Schmidt, Cogan, & Houang, 2011) have also described teacher quality is a major school-related factor influencing the quality of education in general and students' mathematics achievement in particular. Thus, although the relationship is complicated, it is reasonable to conjecture that mathematics teacher expertise is a major factor affecting student achievement, as teacher expertise in mathematics instruction will affect teachers' teaching performance (Kaiser & Li, 2011).

As such, the questions of what it means to be an expert mathematics teacher and what characteristics an expert mathematics teacher possesses are central. However, teacher expertise takes different forms in different cultures and teachers' working conditions exert a powerful influence on the development of their expertise (Berliner, 2004). Therefore, teaching expertise and the conception of expert teacher are not universal, but culturally and contextually dependent (Berliner, 2001). As mentioned earlier, the working conditions and culture of mainland China are often described as unfavorable. In addition, as a country with more than five thousand years of history, education in China has its own characteristics and traditions (Gu, 2001, 2006). Therefore, it would be reasonable to conjecture that expert teachers in China may have some unique characteristics not shared by teachers from other cultures, which may also apply for normal mathematics teachers.

However, there is a lack of research on teacher expertise in mathematics instruction in general (Li & Kaiser, 2011) and in exploring the conception and characteristics of expert mathematics teachers in the Chinese context in particular, even though decades have passed since Cooney *et al.* (1988) intimated that "it would be interesting to learn how mathematics educators from other cultures define 'expert teachers'" (p. 255). In view of this, the main aim of the present study is to explore how "expert mathematics teacher" is conceptualized in the mainland Chinese context, and what sorts of characteristics such expert mathematics teachers would have. Results of the study would be important for understanding what aspects count as important parts of mathematics teacher quality in this specific high-achieving education system.

1.2 Rationale of the Study

There have been many studies focusing on expert mathematics teachers and their teaching practices, and many characteristics of expert teachers have been identified in Western countries (*e.g.*, Berliner, 1995, 2004; Berliner *et al.*, 1988; Borko & Livingston, 1989; Livingston & Borko, 1990). Recently, there has been an increasing interest in studying expert mathematics teachers within mainland China (*e.g.*, Li & Huang, 2008; Li, Huang, Bao, & Fan, 2008; Li, Huang & Yang, 2011; Li & Ni, 2007; Zhu *et al.*, 2007). Many previous studies on expert teachers compared their behaviors and performances to those of novice or non-expert teachers; however, teaching expertise is not a dichotomous variable (Smith, 1999). Therefore, it might be problematic, or at least unreasonable, to compare teachers at the opposite ends of the continuum of teaching expertise.

To date, there is very little understanding of the nature of teacher expertise in mathematics education (Kaiser & Li, 2011). In particular, there is a lack of knowledge on the conception of expert mathematics teacher since very few previous studies have focused on this. Among the existing studies, those taking social-cultural contexts into account were also limited. However, essentially speaking, mathematics teaching is a cultural activity (Stigler & Hiebert, 1999). That is, the behaviors of mathematics teachers in classrooms are fundamentally influenced by cultural values existing in a specific context (Li & Kaiser, 2011). Therefore, while investigating the conception and characteristics of expert mathematics teachers, cultural values and social influences should be taken into account. Furthermore, some researchers have based their investigations on experimental or simulated tasks, rather than studying expert teachers in natural teaching contexts; as such, more investigations in natural settings are needed. Studies to date in mainland China have mainly focused on elementary school mathematics teachers and/or on some specific teacher attributes, such as mathematics knowledge (Li *et al.*, 2005) or pedagogical content knowledge (Zhu *et al.*, 2007); a systematic investigation of expert mathematics teachers has yet to be conducted. Such a segmented inquiry compromises the nature of expertise in teaching (Smith, 1999); therefore, there is a need to systematically explore the characteristics of expert mathematics teachers working at a certain grade level to provide a more comprehensive picture of expert mathematics teachers in mainland China.

However, what is the conception of expert mathematics teachers in mainland China? It is difficult to give a general definition of expert mathematics teachers (Berliner, 2004). This study has adopted Sternberg and Horvath's (1995) prototype view of teaching expertise to investigate the conception of expert mathematics teachers. In addition, characteristics shared by expert mathematics teachers in this context will also be identified and explored. Sociocultural theory is adopted to establish a link between influences of social and cultural contexts on the conception and characteristics of expert mathematics teachers. According to sociocultural theory, self-organized (voluntary) attention, categories perception, conceptual thinking and logical memory vary historically and across different cultures (Gredler & Shields, 2008). In this sense, the conception and characteristics of expert mathematics teachers are context dependent.

1.3 Research Questions of the Study

The major objective of this study is to explore how "expert mathematics teacher" is conceptualized and what characteristics are shared by expert mathematics teachers in Mainland China, which has been regarded as a high-achieving education system in international comparative studies. In particular, three research questions are investigated in this study:

1) How is "expert mathematics teacher" conceptualized by mathematics educators in mainland China?
2) What are the characteristics of expert mathematics teachers in mainland China?
3) How do Chinese social and cultural contexts influence the conception and characteristics of expert mathematics teacher?

1.4 Significance of the Study

This study makes, at a minimum, four significant contributions to the research field of teacher expertise. First, its findings allow those who interested in teacher expertise to develop a deeper understanding of the conception and characteristics of expert teachers in relation to a specific cultural background and a specific subject and from a prototype perspective. Moreover, characteristics shared by expert mathematics

teachers in natural teaching contexts are richly described, and the findings are interpreted with reference to the specific cultural values found where this study was conducted. In this sense, this study helps to clarify what kind of teachers should be regarded as expert and what characteristics they might share in mainland China. More important, the findings provide information about how the cultural and social contexts influence the conception and characteristics of expert mathematics teachers.

Second, the study offers those interested in Chinese mathematics education an opportunity to understand better Chinese mathematics education. This study focuses on mathematics expert teachers in China, a country with students attaining high achievement in IMO, IAEP2, and PISA under unfavorable conditions. It is believed that this study will help readers to understand teacher quality and expertise in mainland China – not only in mathematics, but also in other subjects. Moreover, this study could also provide meaningful information to interpret the excellent achievements of Chinese students in mathematics from the perspective of teacher quality.

Third, the study's findings will be useful for the design of future mathematics teacher education programs. The study offers teacher training program designers a depiction of expert mathematics teachers in real classroom situations, as opposed to the hypothetical or theoretical situations presented in some other studies. The main aim of teacher education programs is to facilitate teachers' professional development; that is, to help teachers, especially pre-service, novice and non-expert teachers, develop their expertise and become, eventually, true experts in their fields (Kaiser & Li, 2011; Leinhardt, 1989; Li & Kaiser, 2011). As such, this study could provide rich information about what constitutes a highly qualified teacher and how such a teacher should be prepared, which has become a hot research topic in the field of mathematics teacher education (*e.g.*, Teacher Education and Development Study in Mathematics (TEDS-M study), Cognitively Activating Instruction (COACTIV study), Mathematics Teaching in the 21st Century (MT21 study)) (Blömeke & Kaiser, 2012; Kunter *et al.*, 2013; Schmidt, Cogan, & Houang, 2011), and help program designers develop more effective mathematics teacher education programs, both in mainland China and elsewhere.

Fourth, the findings offer in-service mathematics teachers a benchmark for their own further improvement. The prototypical conception of expert mathematics teacher identified in the study and rich descriptions of the characteristics of mathematics expert teachers provide a model for teachers to develop their professional skills. In addition, understanding how expert mathematics teachers construct lessons, manage teaching content, and interact with students may assist prospective and beginning mathematics teachers to develop and to overcome the difficulties they might encounter (Livingston & Borko, 1990). Therefore, the findings may facilitate the professsional growth of mathematics teachers at different development stages.

1.5 Outline of the Study

This study consists of nine chapters. The second chapter starts with discussion of three views of concepts, and then discusses the prototype view of teaching expertise, which is the theoretical perspective adopted in this study. The chapter's second section introduces and justifies the adoption of the sociocultural theory in general, and mediation theory in particular, as the theoretical underpinning of this study. The final section of the chapter reviews relevant literature directly related to this study.

Chapter 3 describes the overall social and cultural background of education in mainland China, including teachers' role in Chinese culture, the history of Chinese teacher education system, the teacher qualification and promotion system, the system of basic education and assessment, and the history of mathematics education and curriculum system.

Chapter 4 introduces the research methodology of the study. It first justifies the qualitative nature of this study, and then briefly describes the research design, research site, and participants' information. After this, the data collection and analysis methods are introduced. Strategies to enhance the trustworthiness of the findings are further described at the end of this chapter.

Chapter 5 recounts how the study's 21 interviewees conceptualize "expert mathematics teacher" in terms of her/his knowledge, ability, and traits. Chapter 6 discusses the common characteristics (beliefs, knowledge, and teaching strategies) shared by three expert mathematics teachers. Chapter 7 focuses on similar characteristics found in the three expert mathematics teachers' teaching practices, such as how they plan

their teaching, deal with teaching materials, carry out lesson plans, and organize and reflect upon their teaching.

Chapter 8 discusses the conception and characteristics of expert mathematics teachers in the Chinese social and cultural context, with particular attention to sociocultural factors at four levels: classroom, school, social and cultural. Chapter 9 presents the study's major findings and insights, and makes suggestions for further research.

Chapter Two

Theoretical Orientations and Literature Review

2.1 Introduction

This chapter discusses the theoretical underpinnings and research framework of this study, and reviews the relevant literature. The first section includes information about a prototype view of teaching expertise, which serves as the theoretical perspective for this study to construct the conception of expert mathematics teachers, and to categorize common characteristics of expert mathematics teachers. The second section discusses sociocultural theory, which serves as a theoretical basis to allow the researcher to make research assumptions, develop a conceptual framework, and discuss findings from a sociocultural perspective. The third section reviews literature on expert teachers related to this study.

2.2 Theoretical Perspective of the Study

2.2.1 Views of concepts

The term "concept" has many common and technical meanings that may differ due to people's different "knowledge representation systems, theories of natural language understanding, perceptual processors, theories of logic and semantics, and psychological accounts of semantic memory" (Cohen & Murphy, 1984, pp. 27-28). As such, a unified definition of the notion is not easily arrived at. Smith (1989) stated that a concept "is a mental representtation of a class or individual and deals with what is being represented and how that information is typically used during the categorization" (p. 502). Similarly, Howard (1987) pointed out that "a concept is a mental representation of a category" (p. 2) and people could place stimuli in this category based on similarities between them. Thus, a concept is normally seen as a mental representation. There are three general views of the notion of concept – the classical, probabilistic, and exemplar views (Medin & Smith, 1984).

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DOI 10.1007/978-3-658-03097-1_2, © Springer Fachmedien Wiesbaden 2014

The classical view assumes that “all instances of a concept share common properties that are necessary and sufficient for defining the concept” (Medin & Smith, 1984, p. 115). In other words, as Cohen and Murphy (1984) stated:

> *…each concept corresponds to a set or collection of entities, in which membership is all-or-none. This tradition may be traced to the Aristotelian view that each concept has a definition characterizing its "essence" and providing necessary and sufficient conditions for concept membership. Membership in a concept is considered to be all-or-none: either an object fulfills all of the conditions in the definition, in which case it is a member, or else it fails some condition(s), in which case it is a nonmember*. (p. 29)

According to the classical view, an instance must have all of a concept’s defining properties to be considered an instance of that concept. That is, instances can be represented by logical conjunctive definitions (Michalski, 1993). The classical view has been criticized for failing to specify defining properties, using unclear cases, unnecessary properties and nested concepts, and for taking family resemblance as a determinant of typicality (Medin & Smith, 1984).

In the probabilistic view, “concepts are represented in terms of properties that are only characteristic or probable of class members” (Medin & Smith, 1984, p. 115). The probabilistic view rejects the notion of defining features; instead, it argues that concepts may be represented in terms of features that are typical or characteristic (Murphy & Medin, 1985). The view was developed mainly by Eleanor Rosch and Carolyn Mervis. They developed the prototype theory of concepts, which has been said to “mark a major shift in psychology away from classical theories of concepts and toward probabilistic ones” (Adajian, 2005, p. 231). Some researchers, such as Michalski (1993), treated the prototype view the same as the probabilistic view.

The core of the prototype theory is that concepts are organized around family resemblances, rather than around features that are individually necessary and jointly sufficient for categorization (Mervis & Rosch, 1981; Rosch, 1975; Rosch & Mervis, 1975). Rosch and Mervis (1975) stated that:

> *…members of a category come to be viewed as prototypical of the category as a whole in proportion to the extent to which they bear a family resemblance to (have attributes which overlap those of) other members of the category. Conversely, items viewed as most prototypical of one category will be those with least family resemblance to or membership in other categories.* (p. 575)

From this perspective, the prototype for a category is composed of the most common attribute values relevant to other members of the category. In other words, concepts are organized around a best example (Rosch, 1978; Rosch & Mervis, 1975). Every category is represented by a single prototype or best example, which is "not necessarily one that was specifically learned, but perhaps an average or ideal example that people extract from seeing real examples" (Murphy, 2002, p. 30). Therefore, the prototype is a collection of characteristic features of a certain category that tend to but need not be shared by other instances of that category (Howard, 1987). A prototype can be represented by a list of attributes generated from several members of a category (Goldstone & Kersten, 2003). In other words, prototype representtations are essentially lists of features (Barsalou, 1992) that are usually found in members in the category (Murphy, 2002).

Once the prototype for a category has been determined, categorization can be predicated by determining how similar an object is to the prototype (Goldstone & Kersten, 2003); prototype theorists "often speak of the prototype as the 'best example' of the category and discuss the process of making category judgments in terms of having the prototype in mind or using the prototype in making comparisons" (Grandy, 1992, p. 118). According to prototype theory, similarity-based categories exhibit a graded structure wherein some category members are better exemplars of the category than are others (Rosch, 1973, 1978). Objects in the same category still probably vary in their typicality and differ in their similarity to the prototype.

Compared with classical concepts, the prototype concepts have the following characteristics:

> *prototype categories lack necessary and sufficient conditions; their members need not be absolutely "in" or "out of" the category*

> *but can be members to greater or lesser degrees; their members display family resemblances in a number of characteristic properties rather than uniformly sharing a few defining properties; and they are organized around "prototypical" exemplars.* (Pinker & Prince, 1999, p. 8)

Although the prototype view overcomes some limitations of the classical view, it has its own problems; specifically, it 1) may not adequately capture all of people's knowledge about concepts, and 2) may be too unconstrained (Medin & Smith, 1984).

The exemplar view proposes that concepts are represented by their exemplars, at least in part, instead of by an abstract summary (Smith & Medin, 1999). In other words, categories of concepts may be represented by individual exemplars rather than by a "unitary description of the class as a whole" (Murphy & Medin, 2000, p. 432). Similar to the prototype view, exemplar concepts also categorize an object by comparing it to known exemplars of the category (Medin & Smith, 1984).

There are some differences between the prototype and exemplar views. Firstly, their approaches to conception representations differ – the former involves listing essential features from a single prototype, while the latter represents concepts by a more than one exemplar. Secondly, the categorization process in the two views is different. In the prototype view, it "involves comparing an item to the prototype representation" (Murphy, 2002, p. 95), while the exemplar view "involves comparing an item to all (or many) such exemplars" (Murphy, 2002, p. 95).

Although the exemplar view also overcomes some limitations of the classical view, it has been criticized for lacking "constraints on what properties enter into concepts or even what constitutes a concept" (Medin & Smith, 1984, p. 119); the view limits neither the properties associated with any exemplar, nor the relations between exemplars included in the same representation (Smith & Medin, 2002).

Each of the three views has advantages and disadvantages. The classical view is relatively fixed because necessary and sufficient conditions are needed to define concepts; therefore, it can best be applied to represent well-defined concepts, such as in the law-like nature of the human physical, biological, or social environment (Loocke, 1999). However, the prototype and exemplar views are relatively loosely structured. As the former organizes concepts around prototypes, "only

characteristic (not necessary or sufficient) features are expected" (Goldstone & Kersten, 2003, p. 606), while in the exemplar view, "a conceptual representation consists of only those actual, individual cases that one has observed" (Goldstone & Kersten, 2003, p. 606). Therefore, the prototype and exemplar views can be best applied at the beginning of concept formation, when specific instances have to be discovered first and will be further generalized.

2.2.2 A prototype approach to teaching expertise

Sternberg and Horvath (1995) proposed using the prototype view to reveal the nature of teaching expertise, as "expertise is best thought of as a prototypical concept, bound together by the family resemblance that experts bear to one another" (p. 16) because "there exists no well-defined standard that all experts meet and that no non experts meet" (p. 9). In addition, as they contended, "it is this resemblance to one another that structures the category 'expert'" (p. 9). According to Sternberg and Horvath, a prototype can represent the central tendency of all the exemplars in its category and can serve as a basis for judgments about category membership. Sternberg and Horvath proposed that "teaching expertise be viewed as a similarity-based category with something like a prototype as its summary representation" (p. 9), and as "a category that is structured by the similarity of expert teachers to one another rather than by a set of necessary and sufficient features" (p. 9). Therefore, a prototype of teacher expertise can serve as the summary representation of a similarity-based category of expertise, since it can represent the central tendency of teachers in the category.

According to Sternberg and Horvath (1995), a prototype view can contribute to the dialogue on expert teaching in the following ways:

> *1) Prototype view allows us to adopt a fuller, more inclusive understanding of teaching expertise without falling into the trap of making everyone a presumptive expert;*
> *2) A prototype view provides a basis for understanding apparent "general factors" in teaching expertise;*
> *3) The prototype view provides a basis for understanding and anticipating social judgments about teaching expertise.* (p. 9)

In this way, teaching expertise can be viewed as "a natural category, structured by the similarity of expert teachers to one another and represented by a central exemplar or prototype" (Sternberg & Horvath, 1995, p. 14) and the picture of expert teaching is broadened, and it becomes possible for researchers to use a smaller number of factors or components to describe expert, and even similarity-based categories are considered inherently fuzzy. In addition, by viewing teaching expertise as a prototype, it is possible to "distinguish experts from experienced non experts in a way that acknowledges (a) diversity in the population of expert teachers, and (b) the absence of a set of individually necessary and jointly sufficient features of an expert teacher" (p. 14).

The prototype view has been increasingly adopted by other researchers (*e.g.*, Li, Huang, & Yang , 2011; Lin, 1999; Smith, 1999; Smith & Strahan, 2004) to explore teaching expertise and is adopted as the theoretical perspective of this study, in particular the feature-based model of similarity-based categorization proposed by Sternberg and Horvath (1995). The intention of this study is to explore the conception of and common characteristics found in expert mathematics teachers in mainland China. A list of features related to the conception of expert mathematics teachers and expert mathematics teachers' teaching practices that will emerge from the collected data. This mandates the use of the prototype view. A simple list of expert mathematics teacher features cannot be deemed necessary and sufficient conditions, as claimed in the classical view, and it is paradoxical to identify an expert mathematics teacher at the very beginning of this study as a *known exemplar*, as would be required were the exemplar view employed.

2.3 Theoretical Underpinnings of the Study

2.3.1 Sociocultural theory

Sociocultural theory, as defined by Ratner (2002), is the field that "studies the content, mode of operation, and interrelationships of psychological phenolmena that are socially constructed and shared, and are rooted in other social artifacts" (p. 9). One of its fundamental claims is that its proper focus is human action (Wertsch *et al.*, 1995). Action here may be both external and internal and the action may be carried out by groups with various sizes or by individuals (*ibid*). The goal of sociocultural

research is to explicate the relationship between human action and the cultural, institutional, and historical contexts in which the action occurs (*ibid*). In other words, as Lantolf (2004) explained, "despite the label 'sociocultural' theory is not a theory of the social or of the cultural aspects of human existence. … it is, rather, … a theory of mind … that recognizes the central role that social relationships and culturally constructed artifacts play in organizing uniquely human forms of thinking" (pp. 30-31).

Sociocultural theory has its roots in the writings of L. S. Vygotsky, in which some primary concepts are established, such as mediation, internalization and the zone of proximal development. This study adopts "mediation" in sociocultural theory as the theoretical underpinning, namely "human mind is *mediated*" (Lantolf, 2004, p. 15, emphasis in original) for an in-depth investigation of the conception of expert mathematics teachers held by mathematics educators, and of the common characteristics shared by expert mathematics teachers in mainland China.

Mediation is the central concept of sociocultural theory, and refers to "the process through which humans deploy culturally constructed artifacts, concepts, and activities to regulate (*i.e.*, gain voluntary control over and transform) the material world or their own and each other's social and mental activities" (Lantolf & Thorne, 2006, p. 79). In other words, the term *mediated* means that individuals master a higher level of behavior through their control of cultural symbols (Gredler & Shields, 2008), including numbers, graphs and, above all, speech and writing, all of which are culturally constructed and are passed on and appropriated (often in modified form) from one generation to another (Lantolf, 1994). They are "simultaneously material and conceptual (or ideal) aspects of human goal-directed activity that are not only incorporated into this activity, but are constitutive of it" (Lantolf & Thorne, 2006, p. 62), and play an "essential role in *shaping* action" (Werstch *et al.*, 1995, p. 22, emphasis in original). In other words, "cultural activity systems and the complexes of symbolic mediation they incorporate are simultaneously the effect and the cause of the design and construction of the architecture of the mind" (Rĭo & Alvarez, 1995, p. 217).

According to Vygotsky, there are two levels of mental functions. One comprises primitive or elementary functions, such as involuntary attention, simple perception and natural memory, which are primarily controlled by one's environment and are biologically determined and