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S. V. Gupta

Viscometry for Liquids

Calibration of Viscometers

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S. V. Gupta

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Dedicated to my wife Mrs. Prem Gupta
To my children, grand children
and great grand children
as They all inspire me to live longer

Preface

This book has been initiated by my former colleague, Mr. Anil Kumar Senior Principle Scientific Officer at National Physical Laboratory, who put to me some questions regarding the calibration of viscometers. I also realised that understanding of viscosity and viscous forces have been a subject matter of interest to research workers in many fields like medicine, oil industry, to physicists, chemists, engineers, fuel technologists and to rheologists.

Absolute measurement of viscosity of liquids even with 0.5 % accuracy is rather a tough and time-consuming job. On the other hand, measurement of viscosity relative to viscosity of some standard has been much easier and uncertainty involved is much less than 0.1 %. Hence it becomes necessary to have liquid of known viscosity. Easy availability of water of known characteristics and density to very high degree of accuracy makes it a correct choice to be the primary standard of viscosity. Therefore, the measurement of viscosity of water has been emphasised. Some scientists have spent a major part of their working life in viscosity measurement of water. For example, Scientists at NIST USA have spent some 20 years to establish the value of viscosity of water. The measurement of viscosity of water by capillary and oscillations viscometers has been discussed in Chap. 8 of the book.

Once the value of viscosity of water is established, the next valid step is to build the viscosity scale. This has been discussed in Chap. 2 of the book. Uncertainty propagated to the nth step of viscosity scale has also been derived. Corrections necessary to apply to viscometer constant of the viscometer when used at different temperatures with a different liquid in establishing the viscosity scale have been discussed in detail.

Keeping the popularity of capillary viscometers in mind turbulent and streamline flow has been given. Expression for kinematic viscosity in terms of efflux time and dimensions of the capillary viscometer is established. It has been observed that the kinetic energy correction given in different documents vary in the numerical factor which causes a lot of confusion. Different numerical factors used in the kinetic energy correction are due to use of incoherent units. This point has been amply discussed in Chap. 1.

The rotating and oscillating viscometers along with the viscometers used in specific fields have been discussed in detail. New trends based on modern physical principles like use of PZT crystal, Optical fibre shear waves and Love waves have

been discussed to meet the inner hunger of the researchers in the field. Availability of commercial viscometers, advantage of specific type viscometers have been discussed for the convenience of the users.

It is my pleasant duty to thank my former colleagues at NPL such as Mr. Anil Kumar who along with Mrs. Reeta Gupta has helped me provide all the literature and scientific papers. I need to gratefully acknowledge the help of Dr. Ashok Kumar who has gone through my write-up and advised me on viscometers based on ultrasonic and shear waves PZT crystals. Nothing will be enough to thank Dr. Habil Claus E. Ascheron my Editor at Springer, without whose help and guidance it would have been impossible to get this book published.

Delhi, India, 14 October, 2013 S. V. Gupta

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Chapter 1
Flow Through Capillary

Abstract With introductory remarks about laminar and turbulent flow, expression for rate of discharge of liquid for a laminar flow has been derived. The expression does contain kinetic energy term. Different confusing numerical factors appearing in various literatures in the kinetic energy term have been explained. SI units of dynamic and kinematic viscosities together with their corresponding CGS units have been given. Kinematic viscosity in terms of efflux time for given volume of liquid is given in terms of efflux time. Corrections applicable to viscometer constants to capillary viscometers due to various parameters have been derived. The parameters are gravity, buoyancy, thermal expansion of viscometer bulb, different temperatures of loading and use, surface tension, temperature coefficient of viscosity etc.

1.1 Types of Flow

1.1.1 Turbulent Flow

The type of fluid flow, in which local velocities and pressures that fluctuate randomly, is known as turbulent flow. In turbulent flow the speed of the fluid at a point is continuously changing both in magnitude and direction. The flow of wind and rivers is generally turbulent, even if the currents are gentle. The air or water swirls and eddies while its overall bulk moves along a specific direction.

Most kinds of fluid flow are turbulent, except for laminar flow at the leading edge of solids moving relative to fluids or extremely close to solid surfaces, such as the inside wall of a pipe, or in cases of fluids of high viscosity (relatively great sluggishness) flowing slowly through small channels. Common examples of turbulent flow are blood flow in arteries, oil transport in pipelines, lava flow, atmosphere and ocean currents, the flow through pumps and turbines, and the flow in boat wakes and around aircraft-wing tips. The velocity profile of different layers in turbulent flow is shown in Fig. 1.1.

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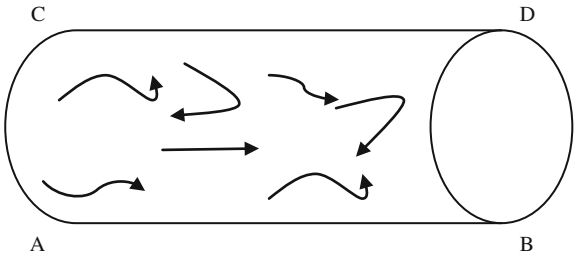

Fig. 1.1 Turbulent flow

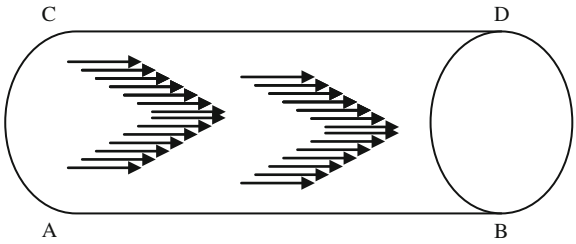

Fig. 1.2 Laminar flow

1.1.2 Laminar Flow

The laminar flow is the type of fluid flow in which the fluid moves in smooth paths or layers. All the fluid is flowing in the same direction. Fluid in contact of the solid stationary objects like the walls of a channel has zero velocity and velocity of consecutive layers increases away from the walls. In a closed channel say a cylindrical pipe the velocity of the layer of fluid along the axis of the pipe is maximum. The liquids of high viscosity moving in relatively small bore tube have the laminar flow. The velocity profile of different layers of liquids with laminar flow in a circular tube is shown in Fig. 1.2.

1.2 Motion in Laminar Flow

In a laminar flow, due to relative motion of layers with respect of each other, a resistive force acts between any two consecutive layers. This resistive force per unit area (shearing stress) divided by rate of change of velocity (shearing strain) is known as coefficient of dynamic viscosity or simply dynamic viscosity. This is normally represented by η.

Let us consider the laminar flow of a liquid along a fixed surface LM, Fig. 1.3. It is experimentally found that a layer CD at a distance $y + dy$ from LM flows faster than the layer AB distance y from the fixed surface LM. If the difference between the velocities of two layers is dv, the velocity gradient along y- the direction perpendicular to the direction of flow is $\frac{dv}{dy}$. Due to this relative motion of

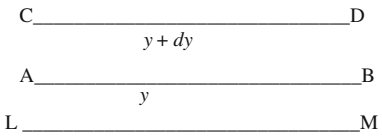

Fig. 1.3 Motion in laminar flow

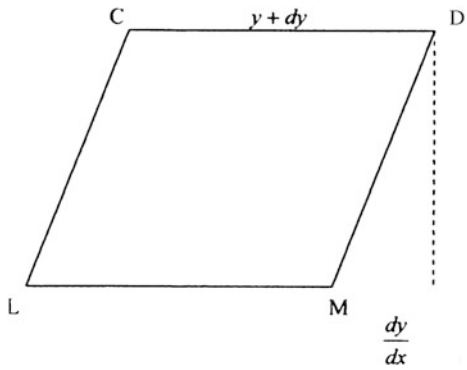

Fig. 1.4 Shearing of the flowing liquid

the layers, resistive force due to internal friction or viscosity acts and whole liquid will shear in the direction of laminar flow and it vertical section will look like as shown in the Fig. 1.4. If F is the force between any two consecutive layers having an area A, then shearing stress is $\frac{F}{A}$ and shearing strain is $\frac{dv}{dy}$. Hence for liquids flowing with the laminar flow, the coefficient of dynamic viscosity η is given as

$$\eta = \frac{F/A}{\frac{dv}{dy}} \tag{1.1}$$

Giving F the frictional force as

$$F = \eta A \frac{dv}{dy} \tag{1.2}$$

This is the Newton's Law of viscous flow for stream line (laminar) flow.

1.3 Unit of Dynamic Viscosity

To obtain the unit of dynamic viscosity (symbol η), write the units of quantities in (1.1), in which it has been defined.

$$\text{SI Unit of dynamic viscosity} = \frac{\text{N/m}^2}{\text{(m/s)/m}} = \frac{\text{pascal}}{1/\text{s}} = \text{Pa·s} \tag{1.3}$$

Similarly the unit of dynamic viscosity η expressed in terms of stress (F/A) and strain

$$\text{(velocity gradient) in CGS system} = \frac{\text{dyn/cm}^2}{\text{(cm/s)/cm}} = \frac{\text{dyn·s}}{\text{cm}^2} = \text{poise} \tag{1.4}$$

The poise is the unit of dynamic velocity in CGS system.

Expressing the CGS units in terms of SI units in (1.4), we get the relation between CGS and SI units.

$$\text{poise} = \frac{10^{-5}\text{N·s}}{10^{-4}\text{m}^2} = 10^{-1}\text{Pa·s} = 0.1\ \text{Pa·s} \tag{1.5}$$

The symbol of CGS unit of dynamic viscosity poise is P, giving

$$1\ \text{P} = 0.1\ \text{Pa·s} \tag{1.6}$$

or

$$10^{-2}\,\text{P} = 1\ \text{cP} = 10^{-3}\text{Pa·s} = \text{mPa·s}$$

$$1\ \text{cP} = 1\ \text{mPa·s} \tag{1.7}$$

In the older literature, the values of dynamic viscosity of different fluids are given in terms of centi-poise symbol (cP). Hence from (1.7) the values of dynamic viscosities in cP would remain unchanged when expressed in millipascal second (mPa·s) a sub-multiple of SI unit.

1.4 Rate of Flow in a Capillary

Assuming that there is a laminar flow that means (1) pressure over any cross-section is constant, (2) there is no radial flow and (3) the liquid in contact of the wall of the circular tube is stationary, we can arrive at a theoretical formula for rate of flow of the liquid.

Referring to Fig. 1.5, let the velocity of the liquid layer at a distance r from the axis of the tube be v, and that of the layer $r + dr$ be $v - dv$, then the velocity gradient will be $-\frac{dv}{dr}$ giving tangential stress from (1.1) as

$$-\eta\frac{\text{dv}}{\text{dr}} \tag{1.8}$$

If the pressure difference between two points, distance L apart, is p, then the driving force on the liquid will be given as

$$\pi r^2 p \tag{1.9}$$

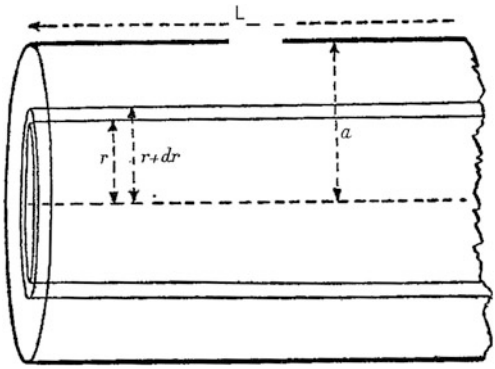

Fig. 1.5 Flow through a capillary

While the viscous force will be

$$-\eta \frac{dv}{dr} 2\pi rL \tag{1.10}$$

Here $2\pi rL$ is surface area of the cylindrical layer at a distance r from the axis. If this entire driving force is utilised in overcoming the viscous force, then

$$p\pi r^2 = -\eta \frac{dv}{dr} 2\pi rL \tag{1.11}$$

Giving

$$\frac{dv}{dr} = -\frac{rp}{2\eta L} \tag{1.12}$$

We know that v is zero at the wall i.e. at $r = a$. Integrating the (1.12) for r between 0 to a, we get

$$\mathrm{v} = \frac{p}{4\eta L}\left(a^2 - r^2\right) \tag{1.13}$$

The velocity profile across the cross-section is, therefore, a parabola.

Here $2\pi rdr$ is the cross-sectional area of the ring bounded by the two layers dr apart and v is the velocity so dQ the rate of flow of the liquid is the product of area of cross-section and velocity v of the liquid. Hence dQ is given by

$$dQ = 2\pi rvdr$$

Substituting the value of v from (1.13) and integrating it with respect of r for limits 0 to a, and, we get

$$Q = 2\pi \frac{p}{4\eta L} \int_0^a r\left(a^2 - r^2\right) dr$$

$$Q = \frac{p\pi a^4}{8\eta L} \tag{1.14}$$

If V is the volume of liquid flown in the time T then

$$\frac{V}{T} = Q = \frac{p\pi a^4}{8\eta L} \tag{1.15}$$

giving η as

$$\eta = \frac{p\pi a^4}{8VL} T. \tag{1.16}$$

1.4.1 Kinetic Energy Correction

In driving (1.11) it is assumed that entire pressure difference p is used for overcoming the viscous forces, however it is a fact that the liquid has acquired a certain velocity hence some pressure difference is required for imparting this velocity. The rate of kinetic energy acquired by the liquid is derived as follows:

Volume flow rate of the liquid between the two layers dr apart and at a distance r from the axis is

$$2\pi r v dr$$

If ρ is the density of the liquid then mass flow rate is given by

$$2\pi\rho r v dr$$

Hence rate of kinetic energy acquired by the liquid $\frac{1}{2}2\pi\rho r v dr v^2$

Using (1.13), total rate of kinetic energy K is given as

$$K = \int_0^{\mathrm{a}} \pi\rho \mathrm{r} \left(\frac{\mathrm{p}}{4\eta \mathrm{L}} \left(\mathrm{a}^2 - \mathrm{r}^2\right)\right)^3 \mathrm{dr}$$

$$\text{Put} \qquad a^2 - r^2 = z$$
$$-2r\,dr = dz$$

The limits of the variable z are a^2 to 0, giving us

$$\begin{aligned} K &= -\int_{a^2}^{0} \frac{\pi\rho}{2} \left(\frac{p}{4\eta L}\right)^3 z^3 dz = \frac{\pi\rho}{2} \left(\frac{p}{4\eta L}\right)^3 \frac{a^8}{4} \\ &= \left(\frac{\pi p a^4}{8\eta L}\right)^3 \frac{\rho}{\pi^2 a^4} = \frac{Q^3\rho}{\pi^2 a^4} \end{aligned} \tag{1.17}$$

The energy spent per second in moving the liquid through capillary is pQ. If p_1 is total external pressure then $p_1 Q$ is total rate of loss of energy giving us

$$p_1 Q = pQ + \frac{Q^3 \rho}{\pi^2 a^4} \tag{1.18}$$

Giving

$$p = p_1 - \frac{Q^2 \rho}{\pi^2 a^4} \tag{1.19}$$

Substituting this value of p in (1.16) and replacing Q by $\frac{V}{T}$, we get

$$\begin{aligned} \eta &= \frac{\pi a^4}{8VL} T \left[p_1 - \frac{V^2 \rho}{T^2 \pi^2 a^4} \right] \\ \eta &= \frac{\pi p_1 a^4}{8LV} T - \frac{V\rho}{8\pi L} \frac{1}{T} \end{aligned} \tag{1.20}$$

In driving (1.20), the energy required in only for the axial flow, however there is some radial flow also, so we use a coefficient say m to compensate for radial kinetic energy. Hence more complete equation is

$$\eta = \frac{\pi p_1 a^4}{8LV} T - \frac{V\rho}{8\pi L} \frac{m}{T} \tag{1.21}$$

In case of a capillary viscometer

$$p_1 = h\rho(1 - \sigma/\rho)g \tag{1.22}$$

h being the fall of the menisci when fixed volume of the liquid has flown through the capillary viscometer.

Substituting the value of p_1 from (1.22) and dividing both sides of (1.21) by ρ, we get

$$\frac{\eta}{\rho} = \frac{\pi h(1 - \sigma/\rho) g a^4}{8VL} T - \frac{mV}{8\pi L} \frac{1}{T} \tag{1.23}$$

For a given viscometer $\frac{V}{8\pi L}$ is constant, so writing B for it, and C for $\frac{\pi h a^4 (1-\sigma/\rho)g}{8LV}$, (1.23) may be written as

$$\frac{\eta}{\rho} = CT - \frac{mB}{T} \tag{1.24}$$

In earlier days m was taken as constant however Cannon and Manning [1] proved that m is not constant. For a trumpet shaped capillary ends m can be represented best as

$$m = 0.037(\mathrm{Re})^{1/2} \tag{1.25}$$

Multiplying factor is 0.037 for trumpet shape ends of the capillary tube, for square cut ends this factor is more than 0.037. However capillary tubes with square

cut ends are not recommended as there is no specific advantage of the use of such tubes but with a distinct disadvantage of having more kinetic energy correction.

Here *Re* is the Reynolds number given by

$$Re = \frac{\rho v L_1}{\eta} \tag{1.26}$$

Here
ρ is the density of the liquid
v is the average velocity
L_1 is distance over which velocity is averaged out and
η is the dynamic viscosity.

In the case of flow through a capillary, average velocity $v = \frac{Q}{\pi D^2/4} = \frac{4V}{\pi D^2 T}$ and the L_1 in (1.26) is the diameter D as the average is taken over the diameter.

Substituting the values of v and L_1 in the definition of Reynolds number, (1.25) becomes

$$m = 0.037 \left[\frac{4VD\rho}{\pi D^2 T \eta}\right]^{1/2} \tag{1.27}$$

For a properly made viscometer the second term in (1.24) is only 3 % of the first term, hence replacing η/ρ by CT in (1.27) will cause an error of not more than 0.09 %.

Hence (1.27) becomes

$$m = 0.074 \left[\frac{V}{\pi CDT^2}\right]^{1/2} \tag{1.28}$$

The second term in (1.24) becomes

$$\begin{aligned} \frac{mB}{T} &= \frac{1}{T} 0.074 \left[\frac{V}{\pi CD}\right]^{1/2} \frac{1}{T} \frac{V}{8\pi L} \\ &= \frac{0.074 V^{3/2}}{8\pi^{3/2} L (CD)^{1/2}} \frac{1}{T^2} \end{aligned} \tag{1.29}$$

$$\frac{mB}{T} = \frac{0.00166 V^{3/2}}{L(CD)^2} \cdot \frac{1}{T^2} \tag{1.30}$$

Substituting the value of $\frac{mB}{T}$ from (1.29) in (1.24), we get

$$\frac{\eta}{\rho} = CT - \frac{E}{T^2} \tag{1.31}$$

Here

$$E = \frac{0.00166V^{3/2}}{L(CD)^{1/2}} \tag{1.32}$$

But $\frac{\eta}{\rho}$ is the kinematic viscosity of the liquid with symbol v. So the v can be expressed as

$$v = CT - \frac{E}{T^2} \tag{1.33}$$

Here

$$C = \frac{\pi h a^4 (1 - \sigma/\rho)g}{8LV} \quad \text{and E} = \frac{0.00166V^{3/2}}{L(CD)^{1/2}} \tag{1.34}$$

It can be easily seen that the dimension of each term on right hand side of (1.33) is (Length)^2/time. Hence SI unit of v is m^2 s^{-1}, of C is m^2 s^{-2} and that of E is m^2 s.

It has been noticed that the numerical factor of E has been given different in various national and International documents. The reason is the use of mixed system of units. The value of the multiplying factor for E in (1.32) is 0.00166 when all the quantities are expressed in coherent units of measurement. Say if V is in m^3, L and D should be in m and C in m^2/s^2 then unit of E is m^{2}s and that of v is m^2/s. Similarly if we take V in cm^3, L and D in cm and C in cm^2/s^2 then unit of E shall be cm^2 s and that v of cm^2/s.

However if V is taken in cm^3, L and D are in cm but C in mm^2/s^2 (mixed system of units) then the factor in E becomes

$$\begin{aligned} \frac{0.00166(10^{-6}\text{m}^3)^{3/2}}{10^{-2}\text{m}(10^{-6}\text{m}^2\,\text{s}^{-2}10^{-2}\text{m})^{1/2}} &= 0.00166\frac{10^{-9}}{10^{-6}}\text{m}^2\,\text{s} \\ &= 0.00166 \cdot 10^{-3} \cdot 10^{6}\text{mm}^2\,\text{s} \\ &= 1.66\ \text{mm}^2\,\text{s} \end{aligned} \tag{1.35}$$

This is the value of the factor given by Fujita et al. in Metrologia paper 2009 [2].

If V is taken in cm^3, L and D are in mm and C in mm^2/s^2, then the factor in E is

$$\begin{aligned} \frac{0.00166(10^{-6}\text{m}^3)^{3/2}}{10^{-3}\text{m}(10^{-6}\text{m}^2\,\text{s}^{-2} \cdot 10^{-3}\text{m})^{1/2}} &= \frac{0.00166 \cdot 10^{-9}\text{m}^{9/2}}{10^{-3}\text{m} \cdot 10^{-9/2}\text{m}^{3/2}\,\text{s}^{-1}} \\ &= 0.00166 \cdot 10^{-3/2}\text{m}^2\,\text{s} \\ &= 0.00166 \cdot 10^{-3/2} \cdot 10^{6}\text{mm}^2\,\text{s} \\ &= 52.5\ \text{mm}^2\,\text{s} \end{aligned} \tag{1.36}$$

ASTM D446-07 [3] has given this value of the factor in E.

1.4.2 End Correction

An additional correction is required to account for the peculiar phenomena at the ends of the tube. It is the work done against the viscous forces in the re-arrangement of velocity distribution. The effect of this is the resistance offered in converging at the entrance and diverging of steam at the exit ends of the tube. The effect is proportional to the radius of the tube and is expressed as a hypothetical addition to the length. The effective length L of the tube then becomes $L + \mathrm{n}a$. For further details of end corrections (change in effective length of the flow tube) one may consult [4]. So finally (1.21) becomes:

$$\eta = \frac{\pi a^4 p_1}{8V(L+na)}T - \frac{m\rho V}{8\pi(L+na)T}. \tag{1.37}$$

1.5 Units of Kinematic Viscosity

By definition the kinematic viscosity of a fluid is the ratio of dynamic viscosity to its density.

Units of dynamic viscosity is

$$\mathrm{Pa{\cdot}s} = \mathrm{kg{\cdot}m^{-1}{\cdot}s^{-2}{\cdot}s} = \mathrm{kg{\cdot}m^{-1}{\cdot}s^{-1}}$$

$$\text{Unit of kinematic viscosity } = \frac{\mathrm{kg{\cdot}m^{-1}{\cdot}s^{-1}}}{\mathrm{kg{\cdot}m^{-3}}} = \mathrm{m^2{\cdot}s^{-1}} \tag{1.38A}$$

But metre square per second is quite big unit so it's sub-multiple namely $\mathrm{mm^2/s}$ is used. For example 1.0034 $\mathrm{mm^2/s}$ is the value of kinematic viscosity of double distilled water at 20 °C.

$$\mathrm{m^2/s} = 10^6 \mathrm{mm^2/s} \tag{1.38B}$$

CGS unit of kinematic viscosity is stoke (St) = $[\{\mathrm{dyne{\cdot}cm^{-2}}\}{\cdot}\mathrm{s}]/\mathrm{g{\cdot}cm^{-3}}$

Expressing the right hand side in SI units, we get

$$= \left[10^{-5}\mathrm{N}/10^{-4}\mathrm{m^2{\cdot}s}\right]/10^3\mathrm{kg/m^3}$$
$$= 10^{-4}\mathrm{m^2/s} \; = 10^2\mathrm{mm^2/s}$$

Giving us

$$1 \text{ centi} - \text{stoke (cSt)} \; = \; 1 \; \mathrm{mm^2/s} \tag{1.39}$$

The values of kinematic viscosities expressed in centi-stokes remain unchanged when quoted in $\mathrm{mm^2 s^{-1}}$.

1.6 Corrections to C Due to Various Parameters

Here we see that C the so called viscometer constant depends upon

1. the local acceleration due to gravity
2. the buoyancy:- densities of liquids and air used
3. the effective value of h which depends on several factors such as the difference in rise of liquid due to different surface tension of the liquids used and the difference in temperatures at which the viscometer was calibrated and used.
4. the volume of liquid V which in turn depends upon the temperature and expansion coefficients of liquid and glass used and essentially affect the effective pressure head h.

1.6.1 Correction Due to Gravity

If a viscometer is calibrated at a place with g_c as the acceleration due to gravity and is used at a place having acceleration due to gravity as g_u then the values of C at the places of calibration and use respectively are given as

$$C_c = \frac{\pi a^4 (1 - \sigma/\rho) g_c}{8LV} \tag{1.40}$$

$$C_u = \frac{\pi a^4 (1 - \sigma/\rho) g_u}{8LV} \tag{1.41}$$

Dividing (1.41) by (1.40), we get

$$C_u = C_c \frac{g_u}{g_c} \tag{1.42}$$

1.6.2 Buoyancy Correction

The c_b is air buoyancy correction on liquids of different density at the time of calibration and use. It is relative to calibration constant of viscometer C_c.

The viscometer constant C determined by calibrating the viscometer, using the standard oil of density ρ_c and with air density σ_c, is expressed as

$$C_c = \frac{\pi a^4 (1 - \sigma_c/\rho_c) g_c}{8LV} \tag{1.43}$$

Similarly when the same viscometer is used to determine the viscosity of an oil of density ρ_u when air density is σ_u, C_u will be expressed as

$$C_{\mathrm{u}} = \frac{\pi a^4 (1 - \sigma_u/\rho_u) g_c}{8LV} \tag{1.44}$$

Dividing the two, we get

$$\frac{C_u}{C_c} = \frac{(1 - \sigma_u/\rho_u)}{(1 - \sigma_c/\rho_c)} = (1 - \sigma_u/\rho_u + \sigma_c/\rho_c)$$

$$C_{\mathrm{u}} = C_{\mathrm{c}}\{1 + (\sigma_c/\rho_c - \sigma_u/\rho_u)\} = C_{\mathrm{c}}(1 + \mathrm{c_b}) \tag{1.45}$$

Giving c_{b} the buoyancy correction relative to C_{c} as

$$c_{\mathrm{b}} = (\sigma_c/\rho_c - \sigma_u/\rho_u). \tag{1.46}$$

1.6.3 Correction Due to Thermal Expansion of Viscometer Bulb

The $\mathrm{c_e}$ is the correction for the effect of thermal expansion of the viscometer due to different temperatures at which the viscometer was calibrated and used for measurement of viscosity of a liquid.

Let volume of the timing bulb be V_{o} at °C then volume V_{c} at the temperature of calibration $\mathrm{t_c}$ will be

$$V_{\mathrm{c}} = V_{\mathrm{o}}(1 + 3\alpha t_{\mathrm{c}}) \tag{1.47}$$

Here α is the coefficient of linear expansion of the material of the viscometer. Similarly V_{u} at the temperature of use (measurement) t_{m} is given by

$$V_{\mathrm{u}} = V_{\mathrm{o}}(1 + 3\alpha t_{\mathrm{m}}) \tag{1.48}$$

Now viscometer constants at the temperatures of calibration and use respectively are:

$$C_c = \frac{\pi a^4 (1 - \sigma/\rho) g_c}{8LV_c} \tag{1.49}$$

$$C_{\mathrm{u}} = \frac{\pi a^4 (1 - \sigma/\rho) h g_c}{8LV_u} \tag{1.50}$$

Substituting the value of $\mathrm{V_u}$ from (1.48) in (1.50) and dividing (1.50) by (1.49) we get

$$C_u = C_c \frac{(1 + 3\alpha t_c)}{(1 + 3\alpha t_m)} \tag{1.51}$$