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Nonlinear Vibrations of Cantilever Beams and Plates

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Chapter 1

Introduction

1.1 Motivation

The beam is one of the fundamental elements of an engineering structure. It finds use in varied structural applications. Moreover, structures like helicopter rotor blades, spacecraft antennae, flexible satellites, airplane wings, gun barrels, robot arms, high-rise buildings, long-span bridges, and subsystems of more complex structures can be modeled as a beam-like slender member. Therefore, studying the static and dynamic response, both theoretically and experimentally, of this simple structural component under various loading conditions would help in understanding and explaining the behavior of more complex, real structures under similar loading.

Interesting physical phenomena occur in structures in the presence of nonlinearities, which cannot be explained by linear models. These phenomena include jumps, saturation, subharmonic, superharmonic, and combination resonances, self-excited oscillations, modal interactions, and chaos. In reality, no physical system is strictly linear and hence linear models of physical systems have limitations of their own. In general, linear models are applicable only in a very restrictive domain like when the vibration amplitude is very small. Thus, to accurately identify and understand the dynamic behavior of a structural system under general loading conditions, it is essential that nonlinearities present in the system also be modeled and studied.

In continuous (or distributed-parameter) systems like structures, nonlinearities essentially couple the linearly uncoupled normal modes, and this coupling could lead to modal interactions (i.e., interaction between the modes), resulting in the transfer of energy among modes. Experiments have demonstrated that sometimes energy is transferred from a directly excited high-frequency mode to a low-frequency mode, which may be extremely dangerous because the response amplitude of the low-frequency mode can be very large compared with that of the directly excited high-frequency mode. A lot of research is under way to understand this and other interesting nonlinear phenomena.

In this dissertation, we study both experimentally and theoretically the nonlinear vibrations of two flexible, metallic cantilever beams under transverse (or external or additive) harmonic excitations. In particular, we investigate the transfer of energy between modes whose natural frequencies are widely spaced – in the absence and presence of an internal resonance. We also develop an experimental parametric identification technique to estimate the linear and nonlinear damping coefficients of a beam along with its effective nonlinearity. In addition, we study experimentally the response of a rectangular, metallic cantilever plate under transverse harmonic excitation.

1.2 Types of Nonlinearity

In theory, nonlinearity exists in a system whenever there are products of dependent variables and their derivatives in the equations of motion, boundary conditions, and/or constitutive laws, and whenever there are any sort of discontinuities or jumps in the system. Evan-Iwanowski (1976), Nayfeh and Mook (1979), and Moon (1987) explain the various types of nonlinearities in detail along with examples. Here, we briefly describe the relevant nonlinearities. In structural mechanics, nonlinearities can be broadly classified into the following categories:

1. *Damping* is essentially a nonlinear phenomenon. Linear viscous damping is an idealization. Coulomb friction, aerodynamic drag, hysteretic damping, etc. are examples of nonlinear damping.

2. *Geometric* nonlinearity exists in systems undergoing large deformations or deflections. This nonlinearity arises from the potential energy of the system. In structures, large deformations usually result in nonlinear strain- and curvature-displacement relations. This type of nonlinearity is present, for example, in the equation governing the large-angle motion of a simple pendulum, in the nonlinear strain-displacement relations due to mid-plane stretching in strings, and due to nonlinear curvature in cantilever beams.

3. *Inertia* nonlinearity derives from nonlinear terms containing velocities and/or accelerations in the equations of motion. It should be noted that nonlinear damping, which has similar terms, is different from nonlinear inertia. The kinetic energy of the system is the source of inertia nonlinearities. Examples include centripetal and Coriolis acceleration terms. It is also present in the equations describing the motion of an elastic pendulum (a mass attached to a spring) and those describing the transverse motion of an inextensional cantilever beam.

4. When the constitutive law relating the stresses and strains is nonlinear, we have the so-called *material* nonlinearity. Rubber is the classic example. Also, for metals, the nonlinear Ramberg-Osgood material model is used at elevated temperatures.

5. Nonlinearities can also appear in the *boundary conditions.* A nonlinear boundary condition exists, for instance, in the case of a pinned-free rod attached to a nonlinear torsional spring at the pinned end.

6. Many other types of nonlinearities exist: like the ones in systems with impacts, with backlash or play in their joints, etc.

It is interesting to note that the majority of physical systems belong to the class of weakly nonlinear (or quasi-linear) system. For certain phenomena, these systems exhibit a behavior only slightly different from that of their linear counterpart. In addition, they also exhibit phenomena which do not exist in the linear domain. Therefore, for weakly nonlinear structures, the usual starting point is still the identification of the linear natural frequencies and mode shapes. Then, in the analysis, the dynamic response is usually described in terms of its linear natural frequencies and mode shapes. The effect of the small nonlinearities is seen in the equations governing the amplitude and phase of the structure response.

1.3 Literature Review

The sheer quantity of material published in the field of nonlinear vibrations of beams makes it almost impossible to list all of them. But the necessary and relevant articles and books will be included here to give a gist of the research done in this area. Unfortunately, the review is restricted only to the English literature.