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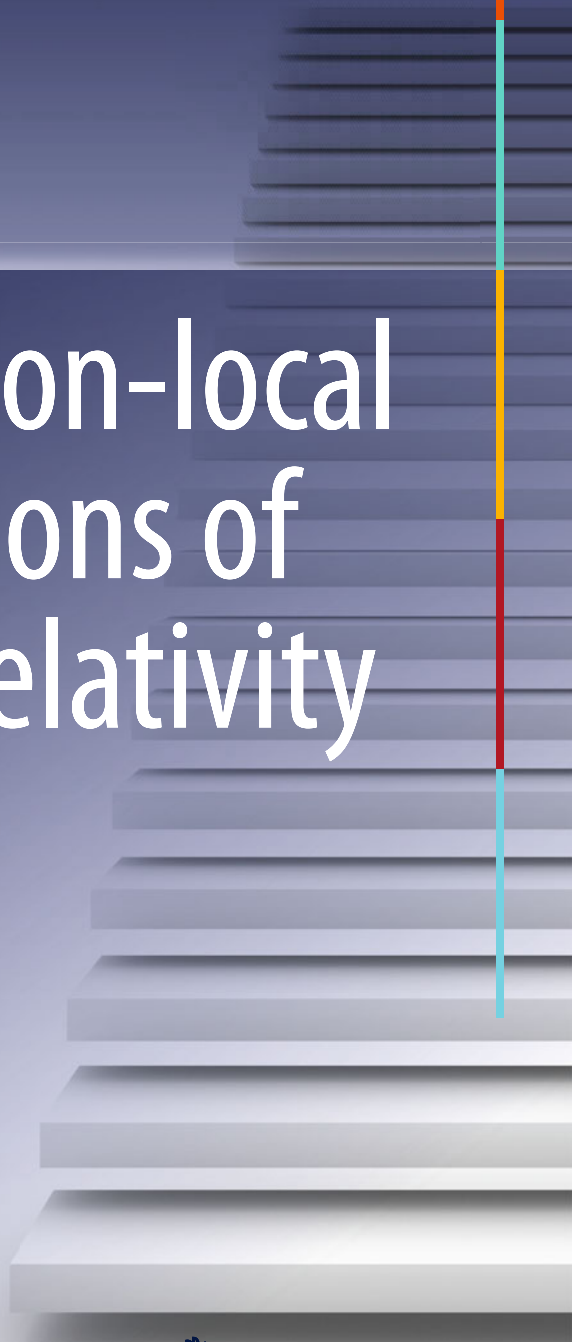
Ermis Mitsou

Infrared Non-local Modifications of General Relativity



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Ermis Mitsou

Infrared Non-local Modifications of General Relativity

Doctoral Thesis accepted by
the University of Geneva, Switzerland

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à Poinpon,
et à tous ceux qui m'ont prêté un
crayon et une feuille quand
j'en avais besoin…

Publications Related to This Thesis

During my Ph.D., the research I have been involved in, within the group of my Ph.D. advisor Michele Maggiore, has focused on several aspects of the problem of dark energy in cosmology. Here are the resulting publications:

- "*Stability analysis and future singularity of the* $m^2 R\Box^{-2} R$ *model of non-local gravity*"
  with Yves Dirian
  JCAP **10** (2014) 065
- "*Cosmological dynamics and dark energy from non-local infrared modifications of gravity*"
  with Stefano Foffa and Michele Maggiore
  Int. J. Mod. Phys. A **29** (2014) 1450116
- "*Apparent ghosts and spurious degrees of freedom in non-local theories*"
  with Stefano Foffa and Michele Maggiore
  Phys. Lett. B **733** (2014) 76–83
- "*A non-local theory of massive gravity*"
  with Maud Jaccard and Michele Maggiore
  Phys. Rev. D **88** (2013) 044033
- "*Bardeen variables and hidden gauge symmetries in linearized massive gravity*"
  with Maud Jaccard and Michele Maggiore
  Phys. Rev. D **87** (2013) 044017
- "*Zero-point quantum fluctuations in cosmology*"
  with Lukas Hollenstein, Maud Jaccard and Michele Maggiore
  Phys. Rev. D **85** (2012) 124031
- "*Early dark energy from zero-point quantum fluctuations*"
  with Lukas Hollenstein, Maud Jaccard and Michele Maggiore
  Phys. Lett. B **704** (2011) 102–107

An important part of this work consisted in the construction and study of a non-local theory of massive gravity and related non-local modifications of General Relativity that would produce a dark energy effect in accordance with observations. This is the subject on which I chose to focus my Ph.D. thesis. A large part of the material presented here is contained in the above publications, but there is a considerable amount of original work as well.

Supervisor's Foreword

In quantum field theory (QFT) we have got used to the fact that our theories, as successful as they might be in the comparison with existing experiments, are only effective theories that will eventually have to be modified at sufficiently high energy or, in the particle physics jargon, at the 'ultraviolet' (UV). For instance, the Fermi theory of weak interactions is now understood as a low-energy limit of the Standard Model. The Standard Model itself is widely expected to be a low-energy approximation to some more fundamental theory, setting in at some high-energy scale. Recently, any QFT that we use will have to be modified when we reach the Planck scale, where quantum gravity sets in. This process, often referred to as the 'UV completion' of a theory, is by now well understood conceptually, in particular since the work of Wilson in the 1970s, and is at the core of modern quantum field theory.

In more recent years has emerged the idea that gravity might also need modifications at low energies ('in the infrared'). At first sight this might be surprising. Don't we know already the low-energy physics? Isn't the frontier of particle physics and QFT just a high-energy frontier? In fact, the situation changed with the impressive advances in cosmology in the past two decades. In particular, the experimental observation of the accelerated expansion of the Universe in 1998 (for which Perlmutter, Schmidt and Riess were awarded the Nobel Prize in 2011) revealed that the expansion of the Universe at the present epoch is accelerating, due to an energy component generically called 'dark energy'. This showed that, in the deep infrared, i.e. at the extremely large distances that are the realm of cosmology, our fundamental theories might need to be modified.

The simplest explanation for dark energy is just a cosmological constant. Indeed, in the past two decades the corresponding cosmological model, ΛCDM, has gradually become the standard cosmological paradigm. Still, the presence of the cosmological constant raises a number of conceptual issues such as the coincidence problem, and the related fact that a cosmological constant is not technically natural from the point of view of the stability under radiative corrections. Thus, much effort is being devoted to looking for alternatives.

An idea which is particularly fascinating from the theoretical point of view is to try to explain the acceleration of the Universe by modifying general relativity at cosmological scales, without introducing a cosmological constant. This idea started to become popular several years ago with the Dvali–Gabadadze–Porrati (DGP) model, and also underlies the intense activity of the past few years on massive gravity and bigravity (see the Thesis of Lavinia Heisenberg, in this Springer series).

A different approach to infrared (IR) modifications of gravity, which is pursued in the Thesis of Ermis Mitsou, is based on the introduction of non-local terms in general relativity. At the fundamental level, quantum field theory is local. Nevertheless, in many situations non-locality emerges at an effective level. This can happen already classically, when one integrates some fast degrees of freedom to obtain an effective theory for the slow degrees of freedom, or at the quantum level. In particular, quantum loops involving light or massless particles induce non-local terms in the quantum effective action. Non-local terms open up new possibilities for building models that modify gravity in the IR, since in the IR operators such as the inverse d'Alembertian becomes relevant.

In the past few years, in my group at Geneva University, we have proposed and investigated some non-local modifications of general relativity, which appear to be quite interesting from both a conceptual and a phenomenological point of view. There are several aspects that can be explored within this program. In particular: (1) At the conceptual level, the introduction of non-localities raises several non-trivial issues, for instance concerning the number of radiative and non-radiative degrees of freedom, causality, etc. (2) At the phenomenological level, one wishes to identify models that work well, providing a viable cosmology both at the level of background evolution and at the level of cosmological perturbations. (3) Eventually, one must be able to derive the required non-local terms from a fundamental local theory.

Following his interests and natural inclinations, in his PhD thesis Ermis has worked in particular on point (1) above. In Chaps. 2, 3 and 4 of the thesis the reader can find an in-depth discussion of several conceptual issues that arise in the context of non-local theories, as well as a detailed discussion of the formal mathematical structures that appear in this context. Ermis has also contributed to the study of the cosmological aspects of the model, at the background level. The corresponding results are presented in Chap. 5.

More recently, after Ermis completed the work that appears in his PhD thesis and moved toward other problems in GR and cosmology, we have further explored the landscape of viable non-local models and their cosmological consequences with other members of the group and collaborators (G. Cusin, Y. Dirian, S. Foffa, N. Khosravi, M. Kunz, M. Mancarella and V. Pettorino). In particular, we now know that models of the class discussed in this thesis are cosmologically viable not only at the level of background evolution, but also at the level of cosmological perturbations, and fit the cosmological data on CMB, supernovae, baryon acoustic oscillations and structure formation, at a level competitive with ΛCDM (and with the same number of free parameters, with the cosmological constant replaced by the mass scale m that characterizes the non-local models). This further adds to the interest of these models. More recently, tentative advances on point (3) have also been made, with the suggestion that the required non-local terms might emerge from nonperturbative effects in the IR, possibly related to the quantum dynamics of the conformal mode, but these are still directions open for investigation.

Geneva
March 2016

Prof. Michele Maggiore

Abstract

The initial motivation in this thesis is to construct a theory of massive gravity which is invariant under coordinate transformations and does not need an external reference metric. This is possible if one resorts to non-local terms in the action. However, phenomenological constraints then lead us to non-local modifications of General Relativity in which gravity is not necessarily massive, but where the cosmology fits the current observational data.

The dynamical structure of a non-local field theory reveals some subtleties compared to its local counterpart. We therefore start by studying the dynamics of massive gauge theories, linear and local, under various viewpoints, in order to highlight the properties that are not exportable to the non-local case. We conclude the study of linear local massive gauge theories by reformulating them as gauge-invariant non-local theories through the Stückelberg formalism. This constitutes our first step in the area of non-local field theory, even though in this case the non-locality is only apparent and disappears with the appropriate choice of gauge. Nevertheless, the technology we have developed allows us to define a linear massive spin-2 theory which is genuinely non-local and gauge-invariant.

We then propose an interlude in order to discuss in depth the various subtleties of non-local field theory. The first one is that both non-local and causal equations of motion cannot be obtained by applying the standard variational principle to some non-local action, but one can generalize the variational principle in order to achieve this. Second, through a localization procedure which involves integrating in auxiliary fields, we see that the dynamical content of these theories is larger than what one would naively guess. These new fields obey dynamical equations of motion, but their initial conditions are constrained by the choice of definition of our non-local operators in the original theory. This fact implies that we cannot consistently quantize these theories, so the latter can only be interpreted as classical effective field theories.

In most of the non-local models that have been studied these auxiliary fields have negative kinetic energy, so a careful examination of their impact on classical stability is required. The bottom line is that their presence does not necessarily invalidate these classical effective theories, because the divergences can be very slow or even countered by non-local effects.

Having clarified these important points, we come back to the linear non-local spin-2 theory we have constructed and try to extend it non-linearly, i.e. to build a non-local theory of gravity. We follow two distinct procedures: one which is based on a non-local action and another which works at the level of the equations of motion using transverse projectors. Thus, we obtain a class of non-linear models which we constrain phenomenologically. This leaves us with one-parameter extensions of the recently proposed models of Maggiore (M) and Maggiore and Mancarella (MM), which continuously bridge the latter to General Relativity with a cosmological constant.

Recent and complete numerical studies of the M and MM models show that they are both statistically equivalent to ΛCDM, within the error margins of the current data. This fact is highly non-trivial, as these models have the same number of parameters as ΛCDM. It also suggests that the extensions we present here are compatible with observations too, since their phenomenology lies somewhere in between. We finally conclude by studying both numerically and analytically the cosmological background of these models and discuss their phenomenological viability.

Acknowledgments

First, I would like to express my gratitude to my supervisor and thesis director, Michele Maggiore, for offering me the opportunity to learn the profession of research and teaching in theoretical physics. During my doctorate I could always count on his valuable advice and I truly benefited from his experience in all aspects of academic activity, and all this without compromising my freedom in pursuing personal/independent research. On top of his qualities as a researcher, I also greatly appreciate his positive attitude as well as his contagious fascination for the mysteries of physics. I am also grateful to him for having carefully read the present thesis, for his comments and corrections.

Second, I would like to thank the people with whom I have had the privilege to collaborate during these 4 years: Maud Jaccard, Lukas Hollenstein, Stefano Foffa, Yves Dirian and, of course, Michele. Our interaction has been an essential element in my PhD, both through its impact on my education and evolution as a researcher and at the human level. I am lucky to have interacted with all these different personalities which will definitely constitute inspiring examples in my professional life.

To the members of our cosmology group I address a very big thank you for the exchanges, whether of academic order or not, for the “crap-coffee” sessions, for their cheerful nature and for the warm atmosphere they generate in our community. I am particularly grateful to Ruth Durrer, Stefano Foffa and Michele for their influence, advice and also their support in my postdoc hunting. A huge thanks to the group's secretaries, Cécile Jaggi-Chevalley and Francine Gennai-Nicole, for their availability and administrative help, and also to Andreas Malaspinas for his technical assistance, availability and for the chats on the second floor of the école de physique which always lasted more than expected.

It is also an immense pleasure to thank the “hurluberlus” of the Office 205 at the Pavillon de physique I, David Daverio and Yves Dirian, for the good mates they are, the coffee-and-cigarettes breaks that “revolutionized” physics, all these way-too-greasy pizzas we shared and for their sturdy characters. I partially dedicate this thesis to David, for his friendship, complicity and in memory of these never-ending and loud debates on fundamental physics, among other things. This experience would have clearly not been the same without him.

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Finally, my warmest thought goes to my wife, my love, Julie, who has always loved, supported and understood me. I am most grateful to her for all this, as well as for her courage, her passion and that subtle strength which inhabits her. I am happy she has trusted our couple for this first step towards our future. It is a step full of hope despite the uncertainty of the paths to which our ambitions may lead us. This work carries the traces of her energy and this is why it is primarily dedicated to her.

I am of course very grateful to the jury members, Ruth Durrer, Pedro Ferreira, Michele Maggiore and Thomas Sotiriou, for their consideration, for reading and evaluating the present thesis and also for their corrections and suggestions.

Geneva Ermis Mitsou
February 2015

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Chapter 1
Introduction

1.1 Background

In the last decades the field of cosmology has witnessed an effervescence which could be compared to the one that permeated particle physics in the 60's and the 70's, resulting in the birth of the Standard Model (SM). As often in science, it is the development of the experimental/observational branch of the discipline that allows the theoretical research to blossom. Indeed, the important activity in observational cosmology during the last two decades turned the discipline into a precise quantitative science, with more and more satellite, balloon and ground-based missions coming to enrich and refine the data pool. This allowed theorists to converge on a six-parameter concordance model, dubbed "ΛCDM", whose statistical predictions fit the data within the current error bars. These two factors, the rich/accurate data and the theoretical concordance model, constitute a solid basis for modern cosmology. This is still a very active area of research, as many more missions will take place in the future, thus providing more accurate input that will allow discriminating between models.

An important aspect of the concordance model, on top of the fact that it matches observations in a satisfying way, is that it mostly relies on well-understood physics. Indeed, on one side there is General Relativity (GR), which determines the dynamics of space-time in the presence of matter, and on the other hand there is the SM, which determines the content and microscopic dynamics of that matter. It is remarkable that the combination of these two pillars of modern theoretical physics suffices to describe already many aspects of the observed cosmology.

Nevertheless, there are also important parts of the concordance model which still remain unaccounted for from the theoretical point of view. The two outstanding ones in late-time cosmology are referred to as the "dark matter" and "dark energy" problems. These are significant extra elements compared to what GR and the SM alone would predict. They have therefore greatly contributed to the enthusiasm for theoretical cosmology and in setting-up further observational missions.

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