Power Systems

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Power Systems Grounding

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To all my teachers and well-wishers who have helped me grow professionally over the years

Md. Abdus Salam

In loving memory of my beloved mother Mrs. Matiza Begum, a wonderful teacher and a mentor, whose utmost selfless love and prayers for me to Almighty Allah have put me in a position where, I'm today. I appreciate the opportunity I've had as a result of the hard work and sacrifice of both of my parents

Quazi M. Rahman

Preface

The book *Power System Grounding* is intended for both lower- and upper-level undergraduate students studying power system, design, and measurement of grounding system, as well as a reference for power system engineers. For reference, this book has been written in a step-by-step method. In this method, this book covers the fundamental knowledge of power, transformer, different types of faults, soil properties, soil resistivity, and ground resistance measurement methods. This book also includes fundamental and advanced theories related to the grounding system.

Nowadays, the demand for smooth, safe, and reliable power supply is increasing due to an increase in the development of residential, commercial, and industrial sectors. The safe and reliable power supply system is interrupted due to different faults including lightning, short circuit, and ground faults. A good grounding system can protect substations, and transmission and distribution networks from these kinds of faults. In addition, a good grounding system ensures the safety of humans in the areas of faulty substations in case of ground faults, and decreases the electromagnetic interference in substations.

This book is organized into seven chapters and two appendices. Chapter 1 deals with the fundamental knowledge of power analysis. Transformer fundamentals and practices are discussed in Chap. 2. Chapter 3 covers the issues on the symmetrical and unsymmetrical faults. Chapter 4 includes the grounding system parameters and resistance. In this chapter, different parameters related to the grounding system and expressions of resistance with different sizes of electrodes are discussed. Chapter 5 presents ideas on different types of soil, properties of soil, influence of different parameters on soil, current density, and Laplace and Poisson equations and their solutions. Chapter 6 describes different measurement methods of soil resistivity. The grounding resistance measurement methods are discussed in Chap. 7.

This book will offer both students and practice engineers the fundamental concepts in conducting practical measurements on soil resistivity and ground resistance at residential and commercial areas, substations, and transmission and distribution networks.

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Special thanks go to our Ex-MSc Engineering student Luis Beltran, The University of Western Ontario, London, Canada who developed part of the derivation on the measurement of ground resistance.

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Lastly, we express our heartfelt thanks to our respective spouses and children for their continued patience during the preparation of the book.

Contents

Chapter 1
Power Analysis

1.1 Introduction

Electrical power is the time rate of receiving or delivering electrical energy that depends on the voltage and current quantities. In an ac circuit, the current and voltage quantities vary with time, and so the electrical power. Every electrical device such as ceiling fan, bulb, television, iron, micro-wave oven, DVD player, water-heater, refrigerator, etc. has a power rating that specifies the amount of power that device requires to operate. An electrical equipment with high power rating generally draws large amount of current from the energy source (e.g., voltage source) which increases the energy consumption. Nowadays, Scientists and Engineers are jointly working on the design issues of the electrical equipment to reduce the energy consumption. Since in the design stage, power analysis plays a vital role, a clear understanding on power analysis fundamentals becomes a most important prep work for any electrical engineer. This chapter reviews these fundamental concepts that include instantaneous power, average power, complex power, power factor, power factor correction and three-phase power.

1.2 Instantaneous Power

The instantaneous power (in watt) $p(t)$ is defined as the product of time varying voltage $v(t)$ and current $i(t)$, and it is written as [1],

$$p(t) = v(t) \times i(t) \tag{1.1}$$

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Consider that the time varying excitation voltage $v(t)$ for an ac circuit is given by,

$$v(t) = V_m \sin(\omega t + \phi) \tag{1.2}$$

where, ω is the angular frequency and ϕ is the phase angle associated to the voltage source. In this case, the expression of the resulting current $i(t)$ in an ac circuit as shown in Fig. 1.1 can be written as,

$$i(t) = \frac{v(t)}{Z\angle\phi} \tag{1.3}$$

where, $Z\angle\phi$ is the impedance of the circuit in polar form, in which Z is the magnitude of the circuit impedance.

Substituting Eq. (1.2) into Eq. (1.3) yields,

$$i(t) = \frac{V_m \sin(\omega t + \phi)}{Z\angle\phi} = \frac{V_m\angle\phi}{Z\angle\phi} = \frac{V_m}{Z}\angle 0^\circ \tag{1.4}$$

$$i(t) = I_m \sin \omega t \tag{1.5}$$

where, the maximum current is,

$$I_m = \frac{V_m}{Z} \tag{1.6}$$

Substituting Eqs. (1.2) and (1.5) into Eq. (1.1) yields,

$$p(t) = V_m \sin(\omega t + \phi) \times I_m \sin \omega t \tag{1.7}$$

$$p(t) = \frac{V_m I_m}{2} \times 2 \sin(\omega t + \phi) \sin \omega t \tag{1.8}$$

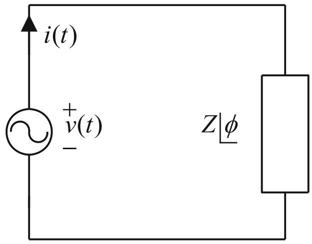


Fig. 1.1 A simple ac circuit

$$p(t) = \frac{V_m I_m}{2}[\cos\phi - \cos(2\omega t + \phi)] \tag{1.9}$$

$$p(t) = \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t + \phi) \tag{1.10}$$

Equation (1.10) presents the resultant instantaneous power.

Example 1.1 An ac series circuit is having the excitation voltage and the impedance $v(t) = 5\sin(\omega t - 25^\circ)$ V and $Z = 2\underline{|15^\circ}\ \Omega$, respectively. Determine the instantaneous power.

Solution

The value of the series current is,

$$I = \frac{5\underline{|-25^\circ}}{2\underline{|15^\circ}} = 2.5\underline{|-40^\circ}\ \mathrm{A}$$
$$i(t) = 2.5\sin(\omega t - 40^\circ)\ \mathrm{A}$$

The instantaneous power can be determined as,

$$p(t) = \frac{5 \times 2.5}{2} \times 2\sin(\omega t - 25^\circ)\sin(\omega t - 40^\circ)$$
$$p(t) = \frac{5 \times 2.5}{2}[\cos 15^\circ - \cos(2\omega t - 65^\circ)]$$
$$p(t) = 6.04 - 6.25\cos(2\omega t - 65^\circ)\ \mathrm{W}$$

Practice problem 1.1
The impedance and the current in an ac circuit are given by $Z = 2.5\underline{|30^\circ}\ \Omega$ and $i(t) = 3\sin(314t - 15^\circ)$ A, respectively. Calculate the instantaneous power.

1.3 Average and Apparent Power

Energy consumption of any electrical equipment depends on the power rating of the equipment and the duration of its operation, and this brings in the idea of average power. The average power (in watt) is defined as the average of the instantaneous power over one periodic cycle, and is given by [2, 3],

$$P = \frac{1}{T}\int_0^T p(t)dt \tag{1.11}$$

Substituting Eq. (1.10) into Eq. (1.11) yields,

$$P = \frac{V_m I_m}{2T} \int_0^T [\cos\phi - \cos(2\omega t + \phi)]\, dt \tag{1.12}$$

$$P = \frac{V_m I_m}{2T} \cos\phi\, [T] - \frac{V_m I_m}{2T} \int_0^T \cos(2\omega t + \phi)\, dt \tag{1.13}$$

$$P = \frac{V_m I_m}{2} \cos\phi - \frac{V_m I_m}{2T} \int_0^T \cos(2\omega t + \phi)\, dt \tag{1.14}$$

The second term of Eq. (1.14) is a cosine wave, and the average value of the cosine wave over one cycle is zero. Therefore, the final expression of the average power becomes,

$$P = \frac{V_m I_m}{2} \cos\phi \tag{1.15}$$

Sometimes, average power is referred to as the true or real power expressed in watts which is the actual power dissipated by the load.

Equation (1.15) can be rearranged as,

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi \tag{1.16}$$

$$P = V_{rms} I_{rms} \cos\phi \tag{1.17}$$

where, V_{rms} and I_{rms} are the root-mean-square (rms) values of the voltage and current components, respectively. The product $V_{rms}I_{rms}$ is known as the apparent power expressed as volt-ampere (VA).

The phase angle between voltage and current quantities associated to a resistive circuit is zero since they are in phase with each other. From Eq. (1.17), the average power associated to a resistive circuit component can be written as,

$$P_R = \frac{V_m I_m}{2} \cos 0^\circ \tag{1.19}$$

$$P_R = \frac{V_m I_m}{2} \tag{1.20}$$

Applying Ohm's law to Eq. (1.20) yields,

$$P_R = \frac{V_m I_m}{2} = \frac{1}{2} I_m^2 R = \frac{V_m^2}{2R} \tag{1.21}$$

The voltage and current quantities associated to either an inductive or a capacitive circuit element are always 90 degree out of phase with each other, and this results in a zero average power (P_L or P_C) as shown in the following expression,

$$P_L = P_C = \frac{V_m I_m}{2} \cos 90^\circ = 0 \tag{1.22}$$

Example 1.2 Figure 1.2 shows an electric circuit with different elements. Determine the total average power supplied by the source and absorbed by each elements of the circuit.

Solution

The equivalent impedance of the circuit is,

$$Z_t = 2 + \frac{3 \times (4 + j6)}{7 + j6} = 4.31\angle 8.48^\circ\ \Omega$$

The value of the source current is,

$$I = \frac{15\angle 45^\circ}{4.31\angle 8.48^\circ} = 3.48\angle 36.52^\circ\ \text{A}$$

The current through the 4 Ω resistor is,

$$I_1 = \frac{3.48\angle 36.52^\circ \times 3}{7 + j6} = 1.13\angle -4.08^\circ\ \text{A}$$

The current through the 3 Ω resistor is,

$$I_2 = \frac{3.48\angle 36.52^\circ \times (4 + j6)}{7 + j6} = 2.72\angle 52.23^\circ\ \text{A}$$

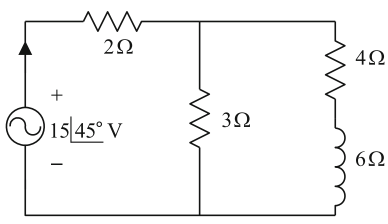
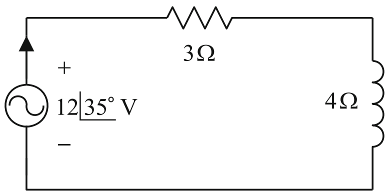


Fig. 1.2 An electrical circuit

Fig. 1.3 A series ac circuit



The average power absorbed by the 3 Ω resistor is,

$$P_1 = \frac{1}{2} \times 2.72^2 \times 3 = 11.1\,\text{W}$$

The average power absorbed by the 4 Ω resistor is,

$$P_2 = \frac{1}{2} \times 1.13^2 \times 4 = 2.55\,\text{W}$$

The average power absorbed by the 2 Ω resistor is,

$$P_3 = \frac{1}{2} \times 3.48^2 \times 2 = 12.11\,\text{W}$$

The average power supplied by the source is,

$$P_4 = \frac{1}{2} \times 15 \times 3.48 \times \cos(45^\circ - 36.52^\circ) = 25.81\,\text{W}$$

The total average power absorbed by the elements is,

$$P_5 = 11.1 + 2.55 + 12.11 = 25.76\,\text{W}$$

Practice problem 1.2
Find the average power for each element of the circuit shown in Fig. 1.3.

1.4 Power Factor

Assuming that the phase angles associated to the voltage and current components are θ_v and θ_i respectively, the average power given in Eq. (1.17) can be written as,

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i) \tag{1.23}$$

From Eq. (1.23) the power factor (pf) can be introduced as,

$$pf = \frac{P}{V_{rms}I_{rms}} = \cos(\theta_v - \theta_i) = \cos\phi \tag{1.24}$$

So, the power factor (pf) is defined as the ratio of the average power to the apparent power associated to a circuit element. Also, based on the mathematical expression in equation (1.50) the power factor can be defined as the cosine of the angle ϕ which is the resultant phase difference between the voltage-phase (θ_v) and the current phase (θ_i) associated to a circuit component. The angle ϕ is often referred to as power factor angle.

The power factor in a circuit element is considered as lagging when the current lags the voltage, whereas, it is considered as leading when the current leads the voltage. In general, industrial loads are inductive and so they have a lagging power factors. A capacitive load has a leading power factor. Every industry always maintains a required power factor by using a power factor improvement unit. It is economically viable for an industry to have a unity power factor or a power factor as close to unity. Few disadvantages of having a load wih low power factor are (i) the kVA rating of the electrical machines is increased, (ii) larger conductor size is required to transmit or distribute electric power at constant voltage, (iii) copper losses are increased, and (iv) voltage regulation is smaller.

1.5 Complex Power and Reactive Power

The product of the rms voltage-phasor and the conjugate of the rms current-phasor associated to an electrical component is known as the complex power (in volt-ampere or VA), and it is denoted by S. Mathematically, it can be written as,

$$\mathrm{S} = V_{rms}I^*_{rms} \tag{1.25}$$

Considering that the phase angles associated to the voltage and current components are θ_v and θ_i respectively, Eq. (1.25) can be written as,

$$\mathbf{S} = V_{rms}\angle\theta_v I_{rms}\angle - \theta_i = V_{rms}I_{rms}\angle\theta_v - \theta_i \tag{1.26}$$

and this becomes,

$$\mathbf{S} = V_{rms}I_{rms}\cos(\theta_v - \theta_i) + jV_{rms}I_{rms}\sin(\theta_v - \theta_i) = P + jQ \tag{1.27}$$

where,

$$P = \mathrm{Re}(\mathbf{S}) = V_{rms}I_{rms}\cos(\theta_v - \theta_i) \tag{1.28}$$

$$Q = \mathrm{Im}(\mathbf{S}) = V_{rms}I_{rms}\sin(\theta_v - \theta_i) \tag{1.29}$$

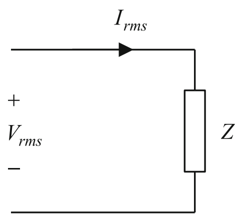


Fig. 1.4 A circuit with impedance

As shown in Eq. (1.27), the real part of the complex power is the real or average power (introduced in the previous section) while the imaginary part of the complex power is known as the reactive power. The reactive power is expressed as volt-ampere-reactive (VAR).

Now, let's discuss the concept of complex power and reactive power with the aid of the above equations, and circuit diagram shown in Fig. 1.4. The impedance of this circuit is given by,

$$\mathbf{Z} = R + jX \tag{1.30}$$

where, R is the resistive component and X is the reactive component of the circuit.

The rms value of the current is,

$$\mathbf{I}_{rms} = \frac{\mathbf{V}_{rms}}{\mathbf{Z}} \tag{1.31}$$

Substituting Eq. (1.31) into Eq. (1.25) yields,

$$\mathbf{S} = \mathbf{V}_{rms}\frac{\mathbf{V}_{rms}^{*}}{\mathbf{Z}^{*}} = \frac{V_{rms}^{2}}{\mathbf{Z}^{*}} \tag{1.32}$$

Again, with the aid of Eq. (1.25), Eq. (1.31) can be rearranged as,

$$\mathbf{S} = \mathbf{I}_{rms}\mathbf{Z}\mathbf{I}_{rms}^{*} = I_{rms}^{2}\mathbf{Z}^{*} \tag{1.33}$$

Substituting Eq. (1.30) into Eq. (1.28) yields,

$$\mathbf{S} = I_{rms}^{2}(R + jX) = I_{rms}^{2}R + jI_{rms}^{2}X = P + jQ \tag{1.34}$$

where, P is the real power and Q is the reactive power, and these quantities are expressed as,

$$P = \mathrm{Re}(\mathbf{S}) = I_{rms}^{2}R \tag{1.35}$$

$$Q = \mathrm{Im}(\mathbf{S}) = I_{rms}^{2}X \tag{1.36}$$

In this case, by comparing Eqs. (1.29) and (1.36), we can conclude that the reactive power is the energy that is traded between the source and the reactive part of the load. It is worth noting that the magnitude of the complex power is the apparent power which has been introduced in the previous section.

Now, let's look at the variation of the complex power in terms of the characteristics of the circuit components. In case of resistive circuit component, the expression of complex power becomes,

$$\mathbf{S}_R = P_R + jQ_R = I_{rms}^2 R \tag{1.37}$$

Here, the real power and the reactive power quantities are,

$$P_R = I_{rms}^2 R \tag{1.38}$$

$$Q = 0 \tag{1.39}$$

The complex power for an inductive component is,

$$\mathbf{S}_L = P_L + jQ_L = jI_{rms}^2 X_L \tag{1.40}$$

Here, the real power and the reactive power quantities are,

$$P_L = 0 \tag{1.41}$$

$$Q_L = I_{rms}^2 X_L \tag{1.42}$$

Similarly, the complex power for a capacitive component is,

$$\mathbf{S}_C = P_C + jQ_C = -jI_{rms}^2 X_C \tag{1.43}$$

Here, the real power and the reactive power quantities are,

$$P_C = 0 \tag{1.44}$$

$$Q_C = I_{rms}^2 X_C \tag{1.45}$$

Now we can introduce another useful power-analysis quantity by dividing Eq. (1.29) by Eq. (1.28) yields,

$$\frac{Q}{P} = \tan(\theta_v - \theta_i) = \tan\theta \tag{1.46}$$

The relationship between the power factor angle to P and Q is known as power triangle. Similarly, we can find the relationship between different components in an impedance which is called impedance triangle. Figure 1.5 shows the power and

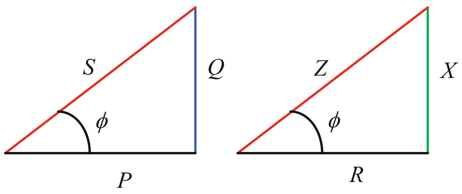


Fig. 1.5 Power and impedance triangles for lagging power factor

impedance triangles for lagging power factor. The following points are noted for different power factors.

$Q = 0$ for resistive loads i.e., unity power factor,
$Q > 0$ for inductive loads i.e., lagging power factor,
$Q < 0$ for capacitive loads i.e., leading power factor.

Example 1.3 Determine the source current, apparent power, real power and reactive power of the circuit shown in Fig. 1.6.

Solution

The rms value of the voltage is,

$$V_{rms} = 5 \times \sqrt{2}\,\underline{|35^\circ} = 7.07\underline{|35^\circ}\text{ V}$$

The value of the inductive reactance is,

$$X_L = 2 \times 4 = 8\,\Omega$$

The value of the circuit impedance is,

$$Z = 5 + j8 = 9.43\underline{|58^\circ}\,\Omega$$

The value of the source current is,

$$I_{rms} = \frac{7.07\underline{|35^\circ}}{9.43\underline{|58^\circ}} = 0.75\underline{|-23^\circ}\text{ A}$$

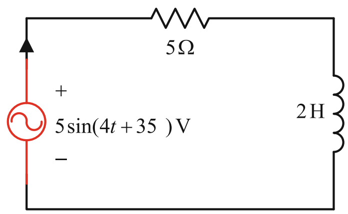


Fig. 1.6 Circuit with resistance and inductance

The value of the complex power is,

$$S = V_{rms}I^{*}_{rms} = 7.07\underline{|35^\circ} \times 0.75\underline{|23^\circ} = 2.81 + j4.5 \text{ W}$$

The value of the apparent power is,

$$P_A = |S| = 5.305 \text{ VA}$$

The value of the real power is,

$$P = \text{Re}(S) = 2.81 \text{ W}$$

The value of the reactive power is,

$$Q = \text{Im}(S) = 4.5 \text{ VAR}$$

Practice problem 1.3
The excitation voltage, resistor and inductance of a series circuit are $v(t) = 6\sqrt{2}\sin(15t + 45^\circ)$ V, 6 Ω and 1.5 H, respectively. Find the source current, apparent power, real power and reactive power.

1.6 Complex Power Balance

Two loads connected in parallel with a voltage source is shown in Fig. 1.7. According to the conservation of energy, the total real power supplied by the source is equal to the sum of the real powers absorbed by the load. Similarly, the total complex power supplied by the source is equal to the sum of the complex powers delivered to each load. Here, the source current can be expressed as,

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 \tag{1.47}$$

Considering that V and I are the rms quantities, the complex power can be written as,

$$\mathbf{S} = \mathbf{VI}^{*} \tag{1.48}$$

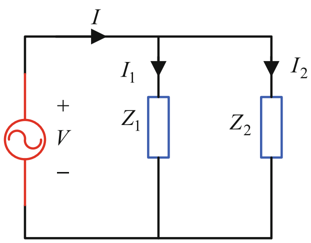


Fig. 1.7 Parallel loads

Substituting Eq. (1.41) into Eq. (1.42) yields,

$$\mathbf{S} = \mathbf{V}[\mathbf{I}_1 + \mathbf{I}_2]^* = \mathbf{V}\mathbf{I}_1^* + \mathbf{V}\mathbf{I}_2^* \quad (1.49)$$

Example 1.4 An ac circuit is shown in Fig. 1.8. Find the source current, power absorbed by each load and the total complex power. Assume that the source voltage is given in rms value.

Solution

The current in the first and second branches are determined as,

$$I_1 = \frac{25}{4} = 6.25\,\text{A}$$

$$I_2 = \frac{25}{3 + j4} = 5\underline{|-53.13^\circ}\,\text{A}$$

Then, the value of the source current is calculated as,

$$I = 6.25 + 5\underline{|-53.13^\circ} = 10.08\underline{|-23.39^\circ}\,\text{A}$$

The value of the complex power for the first branch is,

$$S_1 = VI_1^* = 25 \times 6.25 = 156.25\,\text{W} + j0\,\text{VAR}$$

The value of the complex power for the second branch is,

$$S_2 = VI_2^* = 25 \times 5\underline{|53.13^\circ} = 75\,\text{W} + j100\,\text{VAR}$$

The value of the total complex power is,

$$S = S_1 + S_2 = 231\ \text{W} + j100\,\text{VAR}$$

Which can also be evaluated as,

$$S = VI^* = 25 \times 10.07\underline{|23.38^\circ} = 231\,\text{W} + j100\,\text{VAR}$$

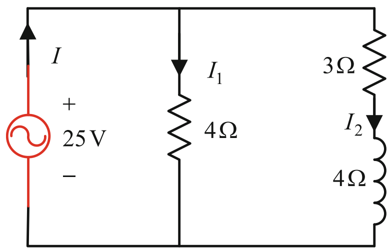


Fig. 1.8 A series-parallel ac circuit

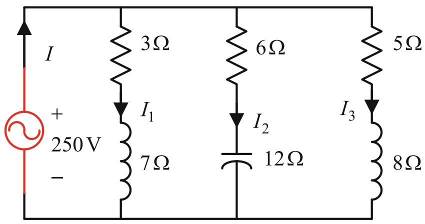


Fig. 1.9 A parallel ac circuit

Practice problem 1.4
Determine the source current, the power absorbed by each load and the total complex power of an electrical circuit shown in Fig. 1.9.

1.7 Power Factor Correction

In the industry, inductive loads draw a lagging current which in turn increases the amount of reactive power. In this case, the kVA rating of the transformer and the size of the conductor should be increased to carry out the additional reactive power. Generally, capacitors are connected in parallel with the load to improve the low power factor by increasing the power factor value. Capacitor draws leading current, and partially or completely neutralizes the lagging reactive power of the load. Consider a single-phase inductive load as shown in Fig. 1.10. This load draws a lagging current I_1 at a power factor of $\cos\phi_1$ from the source.

A capacitor is connected in parallel with the load to improve the power factor as shown in Fig. 1.11. The capacitor will draw current that leads the source voltage by 90°. The line current is the vector sum of the currents I_1 and I_2 as shown in Fig. 1.12. Figure 1.13 shows a power triangle to find the exact value of the capacitor. As shown in equation (na5), the reactive power of the original inductive load can be written as,

$$Q_1 = P\tan\phi_1 \tag{1.50}$$

The main objective of this analysis is to improve the power factor from $\cos\phi_1$ to $\cos\phi_2$ $(\phi_2 < \phi_1)$ without changing the real power. In this case, the expression of new reactive power will be,

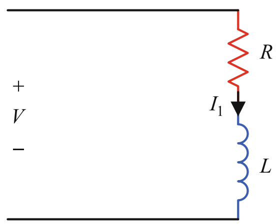


Fig. 1.10 A single-phase inductive load

Fig. 1.11 Capacitor with a parallel inductive load

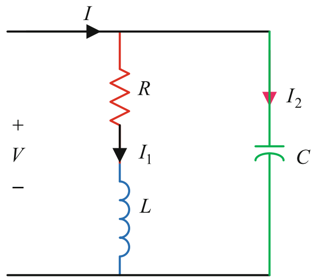


Fig. 1.12 Vector diagram for power factor correction

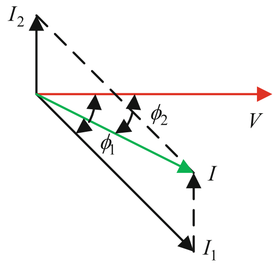
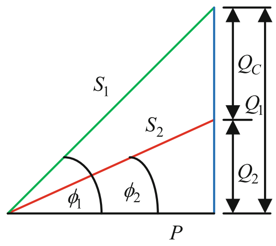


Fig. 1.13 Power triangle with power factor correction



$$Q_2 = P \tan \phi_2 \tag{1.51}$$

The reduction in reactive power due to parallel capacitor is,

$$Q_C = Q_1 - Q_2 \tag{1.52}$$

Substituting Eqs. (1.50) and (1.51) into Eq. (1.52) yields,

$$Q_C = P(\tan \phi_1 - \tan \phi_2) \tag{1.53}$$

The reactive power due to capacitance is,

$$Q_C = \frac{V_{rms}^2}{X_C} = \omega C V_{rms}^2 \tag{1.54}$$

Substituting Eq. (1.54) into Eq. (1.53) yields,

$$Q_C = \omega C V_{rms}^2 = P(\tan\phi_1 - \tan\phi_2) \quad (1.55)$$

$$C = \frac{P(\tan\phi_1 - \tan\phi_2)}{\omega V_{rms}^2} \quad (1.56)$$

Equation (1.56) provides the value of the parallel capacitor.

Example 1.5 A 110 V(rms), 50 Hz power line is connected with 5 kW, 0.85 power factor lagging load. A capacitor is connected across the load to raise the power factor to 0.95. Find the value of the capacitance.

Solution

The value of the initial power factor is,

$$\cos\phi_1 = 0.85$$
$$\phi_1 = 31.79^\circ$$

The value of the final power factor is,

$$\cos\phi_2 = 0.95$$
$$\phi_2 = 18.19^\circ$$

The value of the parallel capacitor can be determined as,

$$C = \frac{5 \times 1000(\tan 31.79^\circ - \tan 18.19^\circ)}{2\pi \times 50 \times 110^2} = 383\ \mu\text{F}$$

Practice problem 1.5

A 6 kW load with a lagging power factor of 0.8 is connected to a 125 V(rms), 60 Hz power line. A capacitor is connected across the load to improve the power factor to 0.95 lagging. Calculate the value of the capacitance.

Example 1.6 A load of 25 kW, 0.85 power factor lagging is connected across the line as shown in Fig. 1.14. Calculate the value of the capacitance when it is connected across the load to raise the power factor to 0.95. Also, Find the power

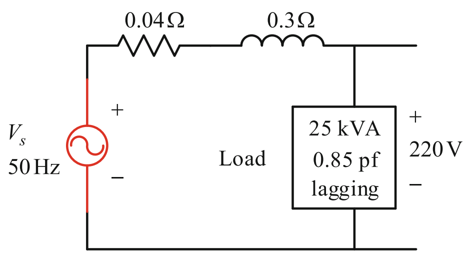


Fig. 1.14 Load with a line

losses in the line before and after the capacitor is connected. Consider that the terminal voltage across the load is constant.

Solution

The initial power factor is,

$$\cos\phi_1 = 0.85$$
$$\phi_1 = 31.78^\circ$$

The final power factor is,

$$\cos\phi_2 = 0.95$$
$$\phi_2 = 18.19^\circ$$

The value of the power of the load is,

$$P = 25 \times 0.85 = 21.25\,\text{kW}$$

The value of the capacitor is,

$$C = \frac{21.25 \times 1000(\tan 31.78^\circ - \tan 18.19^\circ)}{2\pi \times 50 \times 220^2} = 406.62\,\mu\text{F}$$

Before adding capacitor:

The value of the line current is,

$$I_1 = \frac{21250}{0.85 \times 220} = 113.64\,\text{A}$$

The power loss in the line is,

$$P_1 = 113.64^2 \times 0.04 = 516.53\,\text{W}$$

After adding capacitor:

The value of the apparent power is,

$$S = \frac{21250}{0.95} = 22368.42\,\text{VA}$$

The value of the line current is,

$$I_2 = \frac{22368.42}{220} = 101.67\,\text{A}$$

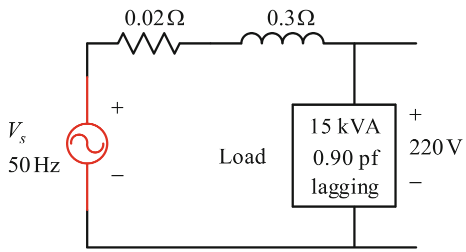


Fig. 1.15 Load with a transmission line

The power loss in the line is,

$$P_2 = 101.67^2 \times 0.04 = 413.51 \text{ W}$$

Practice problem 1.6
A 15 kVA load with 0.95 lagging power factor is connected across the line as shown in Fig. 1.15. Find the value of the capacitance when it is connected across the load to raise the power factor to 0.95. Also, determine the power losses in the line before and after the capacitor is connected. Assume that the terminal voltage across the load is constant.

1.8 Three-Phase System

AC generator generates three-phase sinusoidal voltages with constant magnitude but are displaced in phase by 120°. These voltages are called balanced voltages. In a three-phase generator, three identical coils a, b and c are displaced by 120° from each other. If the generator is turned by a prime mover then voltages V_{an}, V_{bn} and V_{cn} are generated. The balanced three-phase voltages and their waveforms are shown in Figs. 1.16 and 1.17, respectively. The expression of generated voltages can be represented as,

$$V_{an} = V_P \sin \omega t = V_P \angle 0^\circ \tag{1.57}$$

$$V_{bn} = V_P \sin(\omega t - 120^\circ) = V_P \angle -120^\circ \tag{1.58}$$

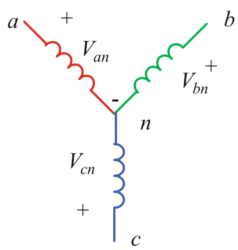


Fig. 1.16 A balanced three-phase system

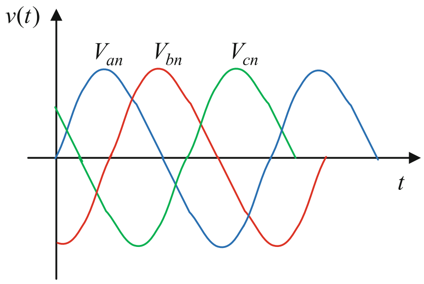


Fig. 1.17 Three-phase voltage waveforms

$$V_{cn} = V_P \sin(\omega t - 240^\circ) = V_P \underline{|-240^\circ} \tag{1.59}$$

Here, V_P is the maximum voltage.

The magnitudes of phase voltages are the same, and these components can be expressed as,

$$|V_{an}| = |V_{bn}| = |V_{cn}| \tag{1.60}$$

1.9 Naming Phases and Phase Sequence

The three-phase systems are denoted either by 1, 2, 3 or *a*, *b*, *c*. Sometimes, three phases are represented by three natural colours namely Red, Yellow and Blue, i.e., *R Y B*. If the generated voltages reach to their maximum or peak values in the sequential order of *abc*, then the generator is said to have a positive phase sequence as shown in Fig. 1.18a. If the generated voltages phase order is *acb*, then the

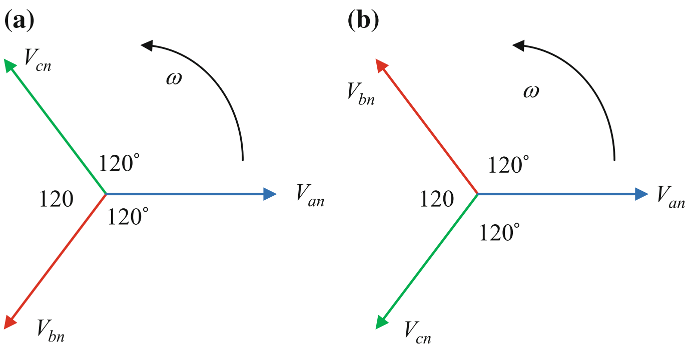


Fig. 1.18 **a** Positive sequence, **b** Negative sequence

generator is said to have a negative phase sequence which is shown in Fig. 1.18b. Here, the voltage, V_{an} is considered as the reference and the direction of rotation is considered as counterclockwise.

In the positive phase sequence, the passing sequence of voltages is given by $V_{an} - V_{bn} - V_{cn}$. In the negative phase sequence, the passing sequence of voltages is $V_{an} - V_{cn} - V_{bn}$.

1.10 Star Connection

The star-connection is often known as wye-connection. A generator is said to be a wye-connected generator when three connected coils form a connection as shown in Fig. 1.19. In this connection, one terminal of each coil is connected to a common point or neutral point n and the other three terminals represent the three-phase supply. The voltage between any line and the neutral point is called the phase voltage and are represented by V_{an}, V_{bn} and V_{cn} for phases a, b and c, respectively. The voltage between any two lines is called the line voltage. Line voltages between the lines a and b, b and c, c and a represented by V_{ab}, V_{bc} and V_{ca}, respectively.

Usually, the line voltage and the phase voltage are represented by V_L and V_P, respectively. In the Y-connection, the points to remember are (i) line voltage is equal to $\sqrt{3}$ times the phase voltage, (ii) line current is equal to the phase current and (iii) current (I_n) in the neutral wire is equal to the phasor sum of the three line currents. The neutral current is zero i.e., $I_n = 0$ for a balanced load.

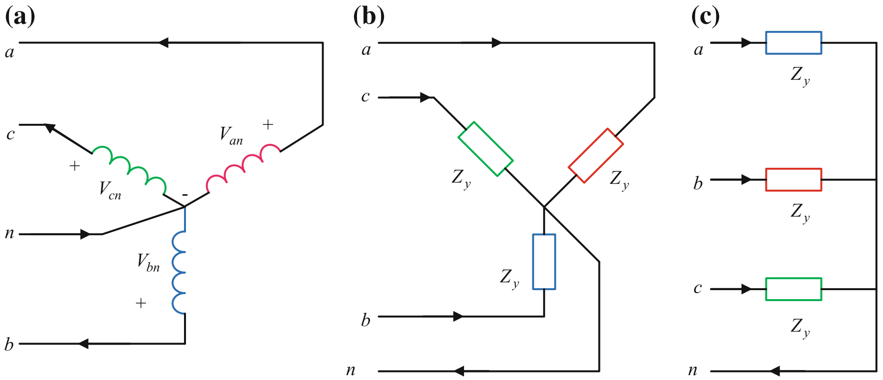


Fig. 1.19 Different types of three-phase wye connection. **a** Generator. **b** Load. **c** Load

1.11 Voltage and Current Relations for Y-Connection

Figure 1.20 shows a three-phase, Y-connected generator. Here V_{an}, V_{bn} and V_{cn} are the phase voltages and V_{ab}, V_{bc} and V_{ca} are the line voltages, respectively. The phase voltages for *abc* phase sequence are,

$$V_{an} = V_p\underline{|0^\circ} \quad (1.61)$$

$$V_{bn} = V_p\underline{|-120^\circ} \quad (1.62)$$

$$V_{cn} = V_p\underline{|120^\circ} \quad (1.63)$$

Apply KVL between lines *a* and *b* to the circuit in Fig. 1.21 and the equation is,

$$V_{an} - V_{bn} - V_{ab} = 0 \quad (1.64)$$

$$V_{ab} = V_{an} - V_{bn} \quad (1.65)$$

Substituting Eqs. (1.61) and (1.62) into Eq. (1.65) yields,

$$V_{ab} = V_P\underline{|0^\circ} - V_P\underline{|-120^\circ} \quad (1.66)$$

$$V_{ab} = \sqrt{3}\, V_P\underline{|30^\circ} \quad (1.67)$$

Similarly, the other line voltages can be derived from the appropriate loops of the circuit as shown in Fig. 1.20. The expressions of other line voltages are,

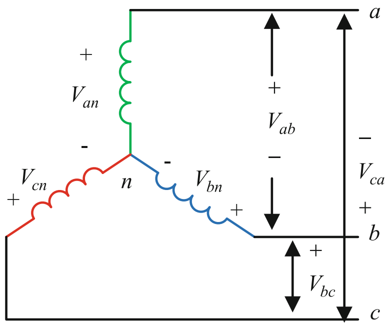


Fig. 1.20 Wye-connected generator

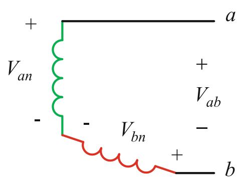


Fig. 1.21 Two lines of wye-connection