

V.I. Ferronsky

# Gravitation, Inertia and Weightlessness

Centrifugal and Gyroscopic Effects of the  
*n*-Body System's Interaction Energy

 Springer

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*To the Memory of  
Sergey Ferronsky*

*and*

*Sergey Denisik  
Colleagues and co-Authors on the Jacobi  
Dynamics Study*

# Preface

Hundreds of works related to the physical meaning and nature of gravitation have been written and their number increases. It means that the problem remains unsolved. Our studies on Jacobi dynamics and, in particular, the recently discovered orbital velocity law of the Solar System's bodies, have allowed the secret of that mysterious phenomenon to be disclosed. It appears that Newtonian gravitation and Galileo's inertia are the centrifugal effects of interaction energy of a self-gravitating  $n$ -body system and its potential field. A self-gravitating celestial body appears to be an excellent natural centrifuge that is rotated by the energy of interacting elementary particles. Dynamical effects of such a centrifuge are the centrifugal and centripetal forces which are taken as the gravity and inertia forces. In every day practice centrifuges are used for separation of the components of matter in gaseous, liquid and solid states with respect to their density (force of weight). In nature, the same forces separate the shells and elementary particles of bodies and their systems. They also provide expansion and creation of bodies and their systems. Fundamentals of Jacobi dynamics completely correspond to the conditions of natural centrifuges. The centrifuge is an excellent experimental model for the study of dynamical effects in solving the many-body problem. In this book, we demonstrate some of those studies.

It was shown in our earlier publications (Ferronsky et al. 1978, 1979a, b, c, 1981a, b, 1982, 1984, 1987, 1996, 2011; Ferronsky and Ferronsky, 2010, 2013) that the needs of farther development of fundamentals in physics and mechanics have appeared for interpretation of new experimental facts obtained by artificial satellites in cosmic space. In fact, it was found by analysis of artificial satellite orbits that the Earth and the Moon do not stay in hydrostatic equilibrium, a conclusion that has been accepted as a basic postulate in the existing theories of their motion and the inner structure. That result means that the applied model of the hydrostatic equilibrium for celestial bodies in a uniform force field is not proved by the observed effects of gravitational mass interaction. The theory of the Earth and other planets' configuration is also based on hydrostatics. In this case, because the sum of the inner forces and moments are equal to zero, the bodies are considered as

solid objects and their rotation is accepted as inertial, which also is not proved by the observation.

A serious discrepancy was found in the motion of Earth and other planets related to the ratio between potential and kinetic energy. It is well known that the potential energy of the Earth exceeds almost by 300 times the kinetic energy presented by inertial rotation of the planets. The same and even higher ratio is valid for the other planets, the Sun and the Moon. But according to the virial theorem the potential energy should be twice as much of kinetic energy. It means that the Earth and other planets exist without kinetic energy. An idea has appeared that there is some latent form of motion of the particles constituting the bodies, which has not been taken into account and is not considered in the existing theories.

Taking into account the relationship between gravitational moments and the gravitational field of the Earth observed by study of artificial satellites of the Earth, we come back to the derivation of the virial theorem in classic mechanics. Replacing the vector forces and moments by their volumetric values, we obtained for an  $n$ -particle system, an understanding of the condition of its dynamical equilibrium in its own force field. The new generalized form of the virial theorem remains in the framework of Newtonian laws of motion but with periodic components expressed by the second derivative from the polar moment of inertia. Thus, for study of a body dynamics in its own force field, the condition of hydrostatic equilibrium by dynamic (periodically oscillating) equilibrium is replaced. In this case the planet's kinetic energy is reanimated by oscillating motion of the interacting particles. And the ratio between the potential and kinetic energy to the classic virial theorem condition has reverted. In addition, a new phenomenon of the nature of gravitation as a dynamical effect of innate energy of the interacting elementary particles appears. On the basis of the obtained results we found that gravitation, inertia and weightlessness have a common innate nature in the form of elementary particles that provide interaction energy, which determines all the dynamical processes in creation and decay of natural systems.

Astrophysical science has proven that the forms of motion of material particles and objects observed in nature are determined by interaction of their constituting elementary particles. More than 300 sub-nuclear particles have been discovered until now. But a strict definition of the term "elementary particle of matter" until now does not exist, because such a particle has not been identified experimentally or theoretically. W. Heisenberg (1966) in his work "Introduction to the unified theory of elementary particles" notes that according to his and other researcher's experimental data, at collision of two particles of high energy, multiple other elementary particles appear. But they do not necessarily appear to be smaller than the colliding particles. Moreover, it appears that the new particles are always born of the same type independently of the nature of the collision. And also, the excess of kinetic energy of the colliding particles is converted into the matter of the created particles. It follows from the observation that the different elementary particles produced can be considered as various forms of existing matter or energy. The size of the new particles remains the same.

Taking into account the above experimental results, the elementary particles in this work are understood as the sub-nuclear material particles, which form the basis for all varieties of objects of the material world.

The most valuable result of our studies is discovery of the new law of planets and their satellites orbiting in the Solar System (Ferronsky and Ferronsky 2013). In this discovery, the astrophysics' postulate about the relationship of motion of the natural objects with interaction of their elementary particles has been proved. The law demonstrates that all planets and satellites have been orbited by the first cosmic velocity of their protoparents. Namely, the planets move in orbits with the first cosmic velocity of the protosun, the radius of which was equal to the semi-major axis of the modern orbit of each planet. The satellites of each planet move with mean orbital velocity equal to the first cosmic velocity of the corresponding planet having radius equal to the semi-major axis of the modern orbit of each satellite. This law holds for all the small planets of the asteroid belt and for all the comets. Theoretically the law follows from solution of Jacobi's virial equation and proved by astronomical observations. It follows from the discovered law that the postulate accepted until now on gravitational attraction of two interacting bodies appears to be a speculation. In fact, the orbital motion is initiated by the outer gravitational field of the central parental body. And the direction of the orbiting is determined by Lenz's rule. Thus, the gravitational field of a celestial body is the centrifugal effect of the body's interacting elementary particles energy and the matter and its energy are the innate natural discrete-wave phenomena. On this basis, we conclude that gravitation and inertia are centrifugal and equal to its centripetal effects of the elementary particles interaction energy leading to redistribution of the particles energy and changes in the body's mode motion. All other dynamical processes should follow from that effect. A self-gravitating body is an excellent example of a natural centrifuge.

The permanently acting process of the elementary particles interaction determines the evolution of a natural body. According to the Archimedes' law, continuous destruction of mass particles and their shell separation with respect to density takes place at their interaction. The upper lighter shell, after its density enriches the state of the weightlessness (relative to the whole body), separates and starts the formation of the secondary body. That is the process of body decay. Its elementary particles collision and scattering are the modes of interaction. The frequency of the particle interaction is the measure of their energy. In the Newtonian theory, that process the straight linear motion with acceleration under outer force action is proposed.

The body shell weightlessness is determined by its state of dynamical equilibrium with the other part of the body. In other words, the weightlessness determines the equilibrium state of the energy pressure between outer gravitational fields of two bodies or two shells. Weightlessness is a consequence of the centrifugal effect of elementary particles interaction that appears at differentiation of a body matter with respect to density. In natural conditions, weightlessness determines the effect of decay of a natural system by its constituting parts or elements at the system expansion. At the system contraction the process of creation of natural objects starts



by creation of mass particles, their aggregates, bodies and galaxies. The equilibrium of larger aggregates here reaches out, gathering interacting particles that have the same frequency of oscillation. This can happen during simultaneous collision of  $n$  particles. Reality of such a process is proved by observation in the galaxy sleeves with almost the same orbital velocities of the stars having different distances from the common center. Those observations appear to be direct evidence of existing large masses of matter which are called “dark matter” and “dark energy”.

In this work, the problem of physical meaning of gravitation, inertia and weightlessness is discussed. On the basis of effects of the new law of the Solar System, bodies orbiting the origin and nature of the above phenomenon are considered. The problem of creation of mass particles and elements from the elementary particles of “dark matter” is analyzed. The basic physics of the Jacobi dynamics from the viewpoint of quantum gravitation and general field theory based on the many body problem solution is discussed.

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# Chapter 1

## Introduction: Phenomenon of Gravitation, Inertia and Weightlessness

**Abstract** A brief story and the physical meaning of forces of gravitation, inertia, weightlessness and reference systems are discussed in this chapter. The theories of gravitation put forth by Newton and Einstein are considered in some detail. This is because, in spite of the criticism and enormous number of alternative versions, the above two theories have remained up to now to be the basis for construction of physical and mathematical models in celestial mechanics, astrophysics, geophysics and global dynamics as a whole. We draw attention to the fact that all the theories are based on the hydrostatic equilibrium of motion. In this connection the majority of researchers dealing with dynamics of the Earth and the planets (Munk and MacDonald, Jeffreys and others) come to the unanimous conclusion that the theories based on hydrostatics do not give satisfactory results in comparison with observations. Some of them straightly say that the theories are incorrect. In any case, the fact of  $j$  initiated this saying on the question about the nature of gravitation that “*I frame no hypotheses*”. In our case, on the basis of the results obtained by studying celestial body motion in the framework of Jacobi dynamics, we come to the conclusion that the point of gravitation determines the integral dynamical effect of elementary particles’ interaction energy which is the innate discrete-wave substance. The problem of inertia forces is most difficult in mechanics because there are too many different classifications depending on accepted reference systems and previous solutions. At some unknown time, a fiction force was introduced as a mathematical base for the D’Alambert principle. Polygamy of the forces is a weak place in mechanics and in different gravitation theories. Newton proposed three main forces that are inertial, impressed and centripetal. The centripetal force has three more varieties like absolute, accelerative and motive. Euler and D’Alambert also posited a number of forces. For such a large number of forces, use of the corresponding mathematical apparatus has to be developed. The vector, tensor, spinor and matrix calculus were developed and the work in that field became continuous. Each of them represents a special scientific direction in mathematically complicating solutions of practical physical, astrophysical and geophysical problems. In scientific literature, the physical meaning of the term “weightlessness” is defined as a complicated state. In relevant encyclopaedias one can find that weightlessness is the state of a material body moving in a gravity field by gravity

forces that do not initiate mutual pressure of the body's particles on each other. The weightlessness effect in cosmic space is compared with man's feelings in the free fall of an elevator. Unfortunately, such a definition of weightlessness contains neither the nature of the unique phenomenon, nor real physical understanding. It is stated in physics that matter in the world, from the elementary particles to the Universe and their force fields, is continuously moving. Absolute rest is impossible. The philosophers say that the motion is the mode of existence of matter and this law is realized by energy. The forms of motion are different in quantity and in quality, and that difference is a subject of scientific and practical interest for human activity. Explanation of the relationship between different forms of motion appears to be the key for understanding a picture of the world development in the framework of the energy conservation law. Gravitation is the most mysterious natural phenomenon in the face of which even science shirks. Modern astrophysics states that the regularities of elementary particles' interaction may open a basic way for understanding laws of motion in the nature. Understanding of those laws is the subject of scientific research. In our case, on the basis of the results obtained by studying celestial body motion in the framework of Jacobi dynamics, we come to the conclusion that the point of gravitation determines the integral dynamical effect of elementary particles' interaction energy which is the innate discrete-wave substance. Let us start our analysis of the existing approaches in studying gravitation with Newtonian gravitation.

## 1.1 Newton's Law of Universal Gravitation

Newton's law of universal gravitation is accepted as one of the fundamental laws of nature ("gravity" is "weight" in Latin). "The world is governed by gravitation", said Newton. Physically this is a philosophical outlook which ancient Greek philosophers started to think about. Kepler has marked in this connection "gravity is a mutual tendency of all bodies". However, only Newton succeeded in formulation of the three laws of motion. On a physical-mechanical basis, he has shown that between any two bodies in the world, the forces of mutual attraction act in accordance with the equation

$$F = \frac{Gm_1m_2}{R^2}, \quad (1.1)$$

where  $G$  is the gravitation constant determined experimentally;  $m_1$ ,  $m_2$  are the bodies' masses;  $R$  is the distance between the bodies;  $F$  is the attraction force.

Passing over from the mass points to the volumetric particles, Newton's law of gravitation leads to his theory of potential, which in the framework of non-relativistic classical physics describes the phenomenon of gravitation. It follows from (1.1) that the masses with density distribution  $\rho(r)$  form the force field as described by Poisson's equation:

$$\Delta\phi = 4\pi\rho, \quad (1.2)$$

where  $\phi$  is the field potential;  $\Delta$  is the Laplacian operator.

The Newtonian field of the potential assumes a long-ranged interaction with an infinite velocity. In this field the gravitating body acquires acceleration:

$$\frac{dr}{dt} = -\text{grad}\phi. \quad (1.3)$$

It means that all the bodies in the force field move with the same acceleration.

The Poisson equation is not disclosed in the structure and mechanism of gravitation. That is why many tens of versions of gravitation theories were written. The mechanism and carrier of attraction forces up to now have not been discovered and the force  $F$  in Eq. (1.1) is the force of weight, but not attraction. Nevertheless, following Newton's definition of dynamical effects of interacted bodies is called "attraction". His famous work "Mathematical principles of natural philosophy", published in 1686, starts with a definition of matter, the quantity of motion and action, the innate, impressed and centripetal forces. Let us recall Newton's original formulations of the more important principles which we cite and discuss later on in the book. For that purpose we quote from the English translation of Newton's *Principia*, made by Andrew Mott in 1929 (Newton 1934).

### **Book I. The Motion of Bodies.**

**Definition I** *The quantity of matter is the measure of the same, arising from its density and bulk conjointly.*

**Definition II** *The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjointly.*

**Definition III** *The vis insita, or innate force of matter, is a power of resisting, by which every body, as much as in it lies, continues in its present state, whether it be rest, or moving uniformly forwards in a right line.*

**Definition IV** *An impressed force is an action exerted upon a body, in order to change its state, either of rest, or of uniform motion in a right line.*

**Definition V** *A centripetal force is that by which bodies are drawn or impelled, or any way tend, towards a point as to a centre.*

*Of this sort is gravity, by which bodies tend to center of the earth; magnetism, by which iron tends to the load stone; and that force, whatever it is, by which the planets are continually drawn aside from the rectilinear motion, which otherwise they would pursue, and made to revolve in curvilinear orbits. A stone, whiled about in a sling, endeavors to recede from the hand that turns it; and by that endeavor, distends the sling, and that with so much the greater velocity, and as soon as it is let go, flier away. That force which opposes itself to this endeavor, and by which the*



*slings continually draws back the stone towards the hand, and retains in its orbit, because it is directed to the hand as the centre of the orbit, I call the centripetal force. And the same thing is to be understood of all bodies, revolved in any orbit. They all endeavor to recede from the centers of their orbits; and were it not for the opposition of a contrary force which restrains them to, and detains them in their orbits, which I therefore call centripetal, would fly off in right lines, with uniform motion...*

*The quantity of any centripetal force may be considered as of three kinds: absolute, accelerative, and motive.*

**Definition VI** *The absolute quantity of a centripetal force is the measure of the same, proportional to the efficiency of the cause that propagates from the centre, through the spaces round about.*

**Definition VII** *The accelerating quantity of a centripetal force is the measure of the same, proportional to the velocity which it generates in a given time.*

**Definition VIII** *The motive quantity of a centripetal force is the measure of the same, proportional to the motion which it generates in a given time.*

*These quantities of forces, we may, for the sake of brevity, call by the names of motive, accelerative, and absolute forces; and for the sake of distinction, consider them with respect to the bodies that tend to the centre of forces towards which they tend; that is to say, I refer the motive force to the body as an endeavor and propensity of the whole towards a centre, arising from the propensities of the several parts taken together; the accelerative force to the place of the body, as a certain power diffused from the centre to all places around to move the bodies that are in them; and the absolute force to the centre, as endued with some cause, without which those motive forces would not be propagated through the space round about; whether that cause be some central body (such as is the magnet in the centre of the magnetic force, or the earth in the centre of the gravity force), or anything else that does not yet appear. For I here design only to give a mathematical notion of those forces, without considering their physical cause and seats...*

*I likewise call attractions and impulses, in the same sense, accelerative and motive; and use the words attraction, impulse, or propensity of any sort towards a centre, promiscuously, and indifferently, one for another; considering those forces not physically, but mathematically: wherefore the reader is not to imagine that by those words I anywhere take upon me to define the kind, or the manner of any action, the causes or the physical reason thereof, or that I attribute forces, in a true and physical sense, to certain centers (which are only mathematical points); when at any time I happen to speak as attracting, or as endued with attractive powers.*

**Law I** *Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.*

**Law II** *The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*

**Law III** *To every action there is always opposite and equal reaction: or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.*

**Section XI. Motion of Bodies Tending to each other with Centripetal Forces.**

Before discussing the problem, Newton essentially notes that “...I approach to state a theory about the motion of bodies tending to each other with centripetal forces, although to express that physically it should be called more correct as pressure. But we are dealing now with mathematics and in order to be understandable for mathematicians let us leave aside physical discussion and apply the force as its usual name”.

**Proposition LVII. Theorem X** *Two bodies attracting each other mutually similar figures about their common centre of gravity, and about other mutually.*

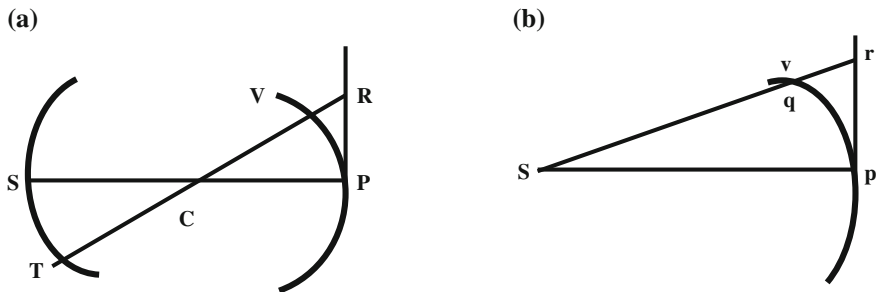
*For the distance of the bodies from their common centre of gravity are inversely as the bodies, and therefore in a given ratio to each other; and hence, by composition of ratios, in given ratio the whole distance between the bodies. Now these distances are carried round their common extremity with uniform angular motion, because lying in the same right line they never change their inclination to each other. But right line that are in a given ratio to each other, and carried round their extremities with an uniform angular motion, describe upon planes, which either rest together with them, or are moved with any motion not angular, figures entirely similar round those extremities. Therefore the figures described by the revolution of those distance are similar.*

**Proposition LVIII. Theorem XI** *If two bodies attract each other with forces of any kind, and revolve about the common centre of gravity: I say, that, by the same forces, there may be described round either body unmoved a figure similar and equal to the figures which the bodies so moving describe round each other.*

*Let the bodies S and P (Fig. 1.1a) revolve about their common centre of gravity C proceeding from S to T, and from P to Q.*

*From the given point s (Fig. 1.1b) let there be continually drawn sp and sq equal and parallel to SP and TQ; and the curve pqv, which the point p described by point p at its revolution will be equal and similar to the curves which are described in its revolution round the fixed point S, will be similar and equal to the curve which the bodies S and P describes about each other; and therefore, by Theor. XX, similar to the curves in curves ST and PQV which the same bodies describe about their common centre of gravity C; and that because the proportions of the lines SC, CP, SP or sp, to each other given.*

**Case 1** *The common centre of gravity C (by Cor. IV of The Laws of Motion) is either at rest, or moves uniformly in a right line. Let us first suppose it at rest, and in s and p let there be placed two bodies, one immovable in s, the other movable in p, similar and equal to the bodies S and P. Then let the right lines PR and pr touch the curves PQ and pq in P and p, and produce CQ and sq to R and r. And because the figures CPRQ, sprq are similar, RQ will be to tq as CP to sp, and therefore in a*



**Fig. 1.1** The problem of two bodies mutually attracted

given ratio. Hence if the force with which the body  $P$  is attracted towards the body  $S$ , and by consequence towards the intermediate centre  $C$ , were to the force with which the body  $p$  is attracted towards the centre  $s$ , in the same given ratio, these forces would in equal times attract the bodies from the tangents  $PR$ ,  $rq$ ; and therefore this last force (tending to  $s$ ) would make the body  $p$  revolve in the curve  $pqv$ , which would become similar to the curve  $PQV$ , in which the first force oblique the body  $P$  to revolve; and their revolutions would be completed in the same times. But because those forces are not to each other in the ratio of  $CP$  to  $sp$ , but (by reason of the similarity and equality of the distance  $SP$ ,  $sp$ ) mutually equal, the bodies in equal times will be equally drawn from the tangents; and therefore that the body  $p$  may be attracted through the greater interval  $rq$ , there is required a greater time, which will vary as the square root of the intervals; because, by Lem. X, the space described at the beginning of the motion are as the square of the times. Suppose, then, the velocity of the body  $p$  to be to the velocity of the body  $P$  as the square root of the ratio of the distance  $sp$  to distance  $cp$ , so that the arcs  $pq$ ,  $PQ$ , which are in a similar proportion to each other, may be described in times that are as the square root to the distance; and the bodies  $P$ ,  $p$ , always attracted by equal forces, will describe round the fixed centres  $C$  and  $s$  similar figures  $PQV$ ,  $pqv$ , the latter of which  $pqv$  is similar and to be figure which the body  $P$  describes round the movable body  $S$ .

**Case 2** Suppose now that the common centre of gravity, together with the space in which the bodies are moved themselves proceeds uniformly in the right line; and (by Cor. VI of *The Laws of Motion*) all the motions in this space will be performed in the same manner as before; and therefore the bodies will describe about each other the same figures as before, which will be therefore similar and equal to the figure  $pqv$ .

**Corollary I** Hence two bodies attracting each other with forces proportional to the square of their distance, describe (by Prop. X), both round their common centre of gravity and round each other, conic sections having their focus in the centre about which the figures are described; and conversely, if such figures are described, the centripetal forces are inversely proportional to the square of the distance.

**Corollary II** *And two bodies, whose focuses are inversely proportional to the square of their distance, describe (by Prop. XI, XII, XIII), both round their common centre of gravity, and round each other, conic sections having their focus in the centre about which the figures are described. And conversely, if such figures are described, the centripetal forces are inversely proportional to the square of distance.*

**Corollary III** *Any two bodies revolving round their common centre of gravity describe areas proportional to the time, by radii drawn both to the centre and to each other.*

**Book II. The Motion of Bodies** *(in resisting medium).*

**Proposition XIX. Theorem XIV** *All the parts of an homogeneous and uniform fluid in any unmoved vessel, and compressed on every side (setting aside the consideration of condensation, gravity, and all the centripetal forces), will be equally pressed on every side, and remain in their places without any motion arising from that pressure.*

**Case 1** *Let a fluid be included in the spherical vessel ABC, and uniformly compressed on every side: I say, that no part of it will be moved by that pressure For it and part, other as D, be moved, all such parts at the same distance from the centre on every side must necessarily be moved at the same time by a like motion; because the pressure of them all is similar and equal; and all other motion is excluded that does not come all of them nearer to the centre, contrary to the supposition*

...

**Proposition XXII. Theorem XVII** *Let the density of any fluid be proportional to the compression, and its parts be attracted downwards by a gravitation inversely proportional to the square of the distances from the centre: I say, that if the distance be taken in harmonic progression, the densities of the fluid at those distances will be in a geometrical progression.*

**Book Three. System of the World** *(in mathematical treatment).*

**Proposition II. Theorem II** *That the forces by which the primary planets are continually drawn off from rectilinear motions, and retained in their orbits, tend to the sun; and are inversely as the squares of the distances of the places of those planets from the sun's centre.*

**Proposition VII. Theorem VII** *That there is a power of gravity pertaining to all bodies, proportional to the several quantities of matter which they contain.*

**Proposition VIII. Theorem VIII** *In two spheres gravitating each towards the other, if the matter in places an all sides round about and equidistant from the centers is similar, the weight of either sphere towards the other will be inversely as the square of the distance between their centers.*

**Proposition IX. Theorem IX** *That the force of gravity, considered downwards from the surface of the planets, decreases nearly in the proportion of the distances from the centre of the planets.*

*If the matter of the planet were of an uniform density, this proportion would be accurate true. The error, therefore, can be no greater than what may arise from the inequality of the distance.*

**Proposition X. Theorem X** *That the motions of the planets in the heavens may subsist an exceedingly long time.*

**Hypothesis I** *That the centre of the system of the world is immovable.*

**Proposition XI. Theorem XI** *That the common centre of gravity of the earth, the sun, and all the planets, is immovable.*

**Proposition XII. Theorem XII** *That the sun is agitated by a continual motion, but never recedes far from the common centre of gravity of all the planets.*

Based on the above proofs, Newton considers the other versions related to the two-body problem which have become basic principles for celestial and classic mechanics.

In Book III, Proposition XIX, Newton considers the problem of the Earth's oblateness as follows:

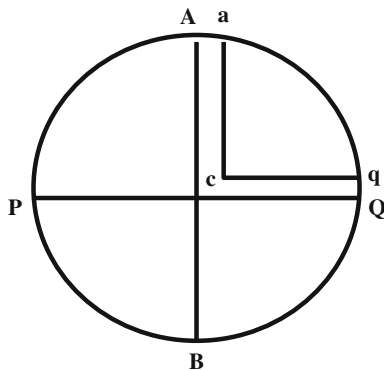
**Proposition XIX. Theorem XIX** *To find the proportion of the axis of a planet to the diameters perpendicular thereto.*

*Our countryman, Mr. Norwood, measuring a distance of 905751 feet of London measure between London and York, in 1635, and observing the difference of latitudes to be  $2^{\circ}28'$ , determined the measure of one degree to be 367196 feet of London measure, that is, 57060 Paris toises. M. Picard, measuring an arc of one degree, and  $22'55''$  of the median between Amiens and Malvoisine, found an arc of one degree to be 57060 Paris toises. M. Cassini, the father, measured the distance upon the meridian from the town Collioure in Roussillon to the observatory of Paris; and his son added the distance from the Observatory to the Citadello of Dunkirk. The whole distance was  $486156\frac{1}{2}$  toises and the difference of the latitudes of Collioure and Dunkirk was 8 degrees, and  $31'11\frac{5}{6}''$ . Hence an arc of one degree appears to be 57061 Paris toises. And from these measures arc conclude that the circumference of the earth is 123249600, and its semidiameter 19615800 Paris feet, upon the supposition that the earth is of a spherical figure.*

Taking advantage of measurements that existed at that time, Newton calculated the ratio of the total gravitation force over the Paris latitude to the centrifugal force over the equator and found that the ratio is equal to 289:1. After that he imagined the Earth in the form of an ellipse of rotation with axis PQ and the channel ACQqca (Fig. 1.2).

If the channel is filled with water, then its weight in the branch ACca will be related to the water's weight in the branch QCcq as 289:288 because of the centrifugal force which decreases the water's weight in the last branch by the unit. He found by calculation that if the Earth has a uniform mass of the matter and has no

**Fig. 1.2** Newton's problem of the Earth oblateness



motion, and the ratio of its axis  $PQ$  to the diameter  $AB$  is 100:101, then the gravity force of the Earth in point  $Q$  relates to the gravity force in the same point of the sphere with radius  $CQ$  or  $CP$  as 126:125. By the same argument the gravity in point  $A$  of a spheroid drawn by revolution around axis  $AB$  relates to the gravity in the same point of the sphere drawn from centre  $C$  with radius  $AC$  as 125:126. However, since there is one more perpendicular diameter, then this relation should be as  $126:125^{1/2}$ . Having multiplied the above ratios, Newton found that the gravity force in point  $Q$  relates to the gravity force in point  $A$  as 501:500. Because of daily rotation, the liquid in the branches should be in equilibrium at a ratio of 505:501. So, the centrifugal force should be equal to  $4/505$  of the weight. In reality the centrifugal force composes  $1/289$ . Thus, the excess in water height under the action of the centrifugal force in the branch  $Acca$  is equal to  $1/289$  of the height in branch  $QCcq$ .

After calculation by hydrostatic equilibrium in the channels, Newton obtained the ratio of the Earth's equatorial diameter to the polar diameter as 230:229, i.e. its oblateness is equal to  $(230-229)/230 = 1/230$ . This result, demonstrating that the Earth's equatorial area is higher than the polar region, was used by Newton for explanation of the observed slower swinging of pendulum clocks on the equator than on the higher latitudes.

At the end of Book III, after discussion of the Moon's motion, the tidal effects and the comets' motion, Newton concludes as follows.

*Hitherto we have explained the phenomena of the heavens and our sea by the power of gravity, but have not yet assigned the cause of this power. This is certain, that it must proceed from a cause that penetrates to very centers of the sun, and planets, without suffering the least diminution of its force, that operates not according to the quantity of the surfaces of the particles upon which it acts (as mechanical causes used to do), but according to the quantity of the solid matter which they contain, and propagates its virtue on all sides to immense distances, decreasing always as the inverse square of the distances. Gravitation towards the sun is made up out of the gravitations towards the several particles of which the body of the sun is composed; and in receding from the sun decreases accurately as*

*the inverse square of the distance as far as the orbit of Saturn, as evidently appears from the quiescence of the aphelion of the planets; nay even to the remotest aphelion of the comets, if those 4 aphelions are also quiescent.*

*But hitherto I have not been able to discover the cause of those properties of gravity from phenomena, and I frame no hypotheses; for whatever is not deduced from the phenomena is to be called an hypothesis; and hypotheses, whether metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy. In this philosophy particular propositions are inferred from the phenomena, and afterwards rendered general by induction Thus it was that the impenetrability, the mobility, and the impulsive force of bodies, and the laws of motion and of gravitation, were discovered. And to us it is enough that gravity does really exist, and act according to the laws which we have explained, and abundantly serves to account for all the motions of the celestial bodies, and of our sea.*

*And now we might add something concerning a certain most subtle spirit which pervades and lies in all gross bodies; by the force and action of which spirit the particles of bodies attract one another at near distances, and cohere, if contiguous; and electric bodies operate to greater distances, as well repelling as attracting the neighboring corpuscles; and light is emitted, reflected, refracted, inflected, and heats bodies; and all sensation is excited, and the members of animal bodies move at the command of the solid filaments of the nerves, from the outward organs of sense to the brain, and from brain into the muscles. But these are things that cannot be explained in few words, nor are we furnished with that sufficiency of experiments which is required to an accurate determination and demonstration of the laws by which this electric and elastic spirit operates.*

Lagrange referred to Newton's work as "*the greatest creature of a human intellect*". It was published in England in Latin in 1686, 1713 and 1725 in his life-time and many times later on. We reiterate that the passages above are from the translation by Andrew Mott in 1729 that was printed in 1934.

As it follows from Newton's definition of the centripetal innate forces, his understanding of their meaning and action in the nature was very wide. The innate force of matter is the power of resistance. It can develop as the force of a body's resistance due to which it remains at rest or moves with constant velocity. It can develop as a body's resistance (reactive) force at outer effect and as a pressure when the body faces an obstacle. In modern mechanics this force is understood synonymously as the force of inertia. The resistance force or force of reaction has found its place in the theory of elasticity, and the pressure is used in hydrodynamics and aerodynamics.

The main meaning of the centripetal force which was introduced by Newton is that each body is attracted to a certain centre. He demonstrates this ability of bodies and objects on the Earth to attract to its geometric centre by action of the gravity force. Newton distinguishes three kinds of manifestation of the centripetal force, namely absolute, accelerating and moving. Absolute value of this force is a measure of the source power of its action from the centre to the outer space. The body's attraction to the centre and emission of attraction from the centre is demonstrated by

Newton in Book III "*The System of the World*", where in Theorem II he notes that gravity forces from the planets are directed to the Sun. In Theorem IX he says that attraction of the planets themselves goes from their surfaces to the centres. According to Newton's idea, the planet's surface is somewhat an area of formation of absolute value of the centripetal force from where it emits up and down.

The accelerating value of the centripetal force by Newton's definition is a measure proportional to velocity which it developed over a long time. The moving value of the centripetal force is a measure proportional to the moment, i.e. to the mass and velocity.

After such a wide spectrum of functions which Newton attributes to the centripetal force, it becomes clear why he was unable to understand its physical meaning and acknowledged: "*But hitherto I have not been able to discover the cause of those properties of gravity from phenomena, and I frame no hypotheses; for whatever is not deduced from the phenomena is to be called an hypotheses; and hypotheses, whether metaphysical or physical, have no place in experimental philosophy. In this philosophy particular propositions are inferred from the phenomena, and afterwards rendered general by induction. Thus it was that the impenetrability, the mobility, and the impulsive force of bodies, and the laws of motion and gravitation, were discovered. And to us it is enough that gravity does really exist, and act according to the laws which we have explained, and abundantly serves to account for all the motions of the celestial bodies, and of our sea*".

It is worth noting that mathematicians, to whom Newton expounded the theory, because of complication in analytical operation with the forces, introduced to celestial mechanics and analytical dynamics the force function, i.e. energy with its ability to develop pressure. Doing so, they practically generalized the physical meaning of the force effects. As to the centripetal forces, later on in Sect. 2.2 of Chap. 2 we shall show that volumetric forces of mass-particle interaction in reality generate Newton's physical pressure which in formulation of practical problems is expressed by the energy. Once more note that Newton, as he expressed himself, instead of correct physical meaning of the concept "*pressure*" gave preference to the concept "*attraction*" to be more understandable to mathematicians.

Newton's problem about the mutual attraction of two bodies, which depict similar trajectories around their common centre of gravity and around each other, is based on the geometric solution of Kepler's problem formulated in his first two laws. Newton's solution is founded on his conception of the centripetal innate forces under which the bodies depict similar trajectories around their common centre of gravity and around each other. In celestial mechanics, developed on the basis of Newton's attraction law, the two-body problem is reduced to an analytical problem of one body, the motion of which takes place in the central field of the common mass. Both Newton's geometric theorem and analytical solution of celestial mechanics are based on the hydrostatic equilibrium state of averaged body motion being brought by Kepler's laws. That fact was well understood by Newton when presenting in detail the hydrostatics laws. However, in both cases the two-body problem was solved correctly in the framework of its formulation. The only difference is that, for Kepler, the planet motion occurs under the action of the



Sun forces, whereas Newton shows that this motion results from the mutual attraction of both the Sun and the planet (Ferronsky and Ferronsky 2010).

In Section V of Book II “*Density and Compression of Fluids: Hydrostatics*” Newton formulates the hydrostatics laws and on their basis in Book III “*The System of the World*” he considers the problem of the Earth’s oblateness, applying real values of the measured distances between a number of points in Europe. Applying the found measurements and hydrostatic approach, he calculated the Earth’s oblateness equal to  $1/230$ , where in his consideration the centrifugal force plays the main contraction effect expanding the body along the equator. In fact the task is related to the creation of an ellipsoid of rotation from a sphere by action of the centrifugal force. Here Newton applied his idea that the attraction of the planet itself goes from the surface to its centre. In this case the total sum of the centripetal forces and the moments is equal to zero and rotation of the Earth should be inertial. It means that the planet’s angular velocity has a constant value.

Inertial rotation of the Earth is accepted a priori. There is no evidence or other form of justification for this phenomenon. There are also no ideas relative to the mode of a planet’s rotation, namely, whether it rotates as a rigid body or there is a differential rotation of separate shells. In modern courses of mechanics there is only analytical proof that, if a body occurs in the outer field of central forces, then the sum of its inner forces and torques is equal to zero. Thus, it follows that the Earth’s rotation should have a mode of a rigid body and the velocity of rotation in time should be constant.

The proof of the conclusion, that if a body occurs in the field of the central forces then the sum of the inner forces and torques is equal to zero, and the moment of momentum has a constant value, is directly related to the Earth’s dynamics (Kittel et al. 1965).

It is known from classical considerations that in the model of two interacting mass points reduced to the common mass centre, which Newton used for the solution of Kepler’s problem having in mind the planets’s motion around the Sun, the inner forces and torques being in the central force field are really equal to zero. The torque, which is a derivative with respect to time from the moment of momentum of material particles of the body, is determined here by the resultant of the outer forces and the planets’ orbits in the central force field entering into the same plane. This conclusion follows from Kepler’s laws of planetary motion.

Passing to the problem of Earth dynamics, Newton had no choice for the formulation of new conditions. The main conditions were determined already in the two-body problem where the planet appeared in the central force field of the reduced masses. The only difference that appears here is that the mass point has a finite dimension. The condition of zero equality of the inner forces and torques of the rotating planet should mean that the motion could result from the forces among which known were only the Galilean inertial forces. Such a choice followed from the inertial motion condition of two-body motion which he already applied. The second part of the problem related to reduction of the two bodies to their common centre of masses and to the accordingly appeared central force, has predetermined the choice of the equation of state. It became the hydrostatic equilibrium of the

body's state being in the outer uniform central force field. The physical conception and mathematical expression of hydrostatic equilibrium of an object based on Archimedes' laws (3d century BP) and the Pascal law (1663) were well known in that time. This is the story of the sphere model with the equatorial and polar channels filled in by a uniform liquid mass in the state of hydrostatic equilibrium at inertial rotation.

In Newton's time the dynamics of the Earth in its direct meaning had not surfaced and it is absent up to now. The planet, rotating as an inertial body and deprived of its own inner forces and torques, has appeared as a dead-born creature. But up to now, the hydrostatic equilibrium condition, proposed by Newton, is the only theoretical concept of the planet's dynamics because it is based on the two-body problem solution which satisfies Kepler's laws and in practice plays the role of Hooke's law of elasticity.

In spite of the noted discrepancies, the problem of the Earth's oblateness was the first step towards the formulation and solution of the highly complicated planet's figure task on which theoretical and experimental study continues up to present time. As to the value of polar oblateness of the Earth, it appears to be much higher. Later observations and measurements show that relative flattening has a smaller value and Newton's solution was needed to have further development.

French mathematician and astronomer A. Clairaut continued Newton's solution of the figure problem of the Earth based on hydrostatics (Clairaut 1947). The degree measurements in the equatorial and northern regions done in the eighteenth century by French astronomers proved Newton's conclusion about the Earth's oblateness, which was at that time regarded with scepticism. However, the measured value of the relative flattening appeared to be different. In the equatorial zone it was equal to  $1/314$ , and in the northern region—to  $1/214$  (Grushinsky 1976). Clairaut himself took part in the expeditions and found that Newton's results are not correct. It was also known to him that the Earth is not a uniform body. Therefore, he focused his strength on taking into account consideration of this effect. Clairaut's model represented an inertia rotating body filled in with liquid having a jumping density. By structure such a model was closer to the real Earth since it had a shell structure. But the hydrostatic equilibrium condition and the inertial rotation as the physical basis for the problem solution were out of hesitation and were taken as before. Clairaut introduced a number of assumptions to the problem formulation. In particular, since the velocity of inertial rotation and the value of the oblateness are small, the boundary areas of the shells and their equilibria were taken as ellipsoidal figures with a common axis of rotation. Clairaut's solution comprised obtaining a differential equation for the shell structured ellipsoid of rotation relative to geometric flattening of its main section.

Proposed by Newton and developed as a Clairaut model of the Earth in the form of a rotating by inertia spheroid filled in with a non-uniform liquid, the mass of which resides in hydrostatic equilibrium in the outer force field, it became generally accepted, commonly used and in principal has not changed up to now. Its purpose was to solve the problem of the planet's figure, i.e. the form of the planet's surface, and this goal in first approximation was reached. Moreover, having been obtained

by a Clairaut equation on surface changes in the acceleration of the gravity force as a function of the Earth's latitude, it opened the way to experimental study of oblateness of the spheroid of rotation by means of measuring the outer gravity force field. Later on, in 1840 Stokes solved the direct and reversed task concerning surface gravity force for a rotating body and over its level applying the known parameters, namely, the mass, radius and angular velocity. The above parameters uniquely determined the gravity force at surface level, which is taken as the quiet ocean's surface, and in all outer space. By that task the relation between the Earth's figure and the gravity force was determined. In the middle of the last century, Molodensky (Molodensky and Kramer 1961) proposed the idea to consider the real surface of the Earth as a reduced surface and solved the corresponding boundary task. The doctrine of the spheroidal figure of the Earth has found common understanding and researchers, armed with theoretical knowledge, started to refine the dimensions and other details of the ellipsoid of rotation and to derive the corresponding corrections.

Earth dynamics was always of interest, not only to researchers of its configuration. Fundamentals of all the Earth, planetary and solar system sciences are defined first of all by the laws of motion of the Earth itself, where the confidence limit of the laws can be checked by observation. Moreover, all the sense of human life is connected with this planet. As far as the techniques and instruments for observation were developed, geodesists, astronomers and geophysicists have noticed that in the planet's inertial rotation, some irregularities and deviations relative to the accepted standard parameters and hydrostatic conditions have appeared. Those irregularities or inaccuracies, as they are often called, a number of which are counted by more than ten, were finally incorporated into two problems, namely, variation of the angular velocity in the daily, monthly, annually and secular time scale, and variation in the poles motion in the same time scales. Just after the problems became obvious and did not find resolution in the frame work of the accepted physical and theoretical conceptions of celestial mechanics, the latter has lost interest in the problems of Earth dynamics. In this connection the well-known German theoreticians in dynamics Klein and Sommerfeld stated that the Earth's mechanics appear to be more complicated than celestial mechanics and represent "*some confused labyrinths of geophysics*" (Klein and Sommerfeld 1903). The geophysicists themselves started to solve their own problems. They had no other way except to search for the causes of the observed inaccuracies. In order to study irregular velocity of the Earth's rotation and the pole motion, numerous projects of observation and regular monitoring were organized by the planetary network. As it was always in such cases, the cause of the observed effects was searched for in the effects of perturbations coming from the Moon and the Sun, and also in the influence of dynamical effects of their own shells like the atmosphere, the oceans and the liquid core, existence of which is justified by many researchers. In some works the absence of the hydrostatic equilibrium in distribution of the masses and strength in the planet's body is considered as the reason of irregular velocity of the Earth's rotation.

Many publications have been devoted to analysis of the observed inaccuracies in the Earth's rotation together with explanation of their possible causes, based on

experimental data and theoretical solutions. The most popular review work in the twentieth century was the book of the known English geophysicist Harold Jeffreys "*The Earth: Its Origin, History and Physical Constitution*". The first publication of the book was in 1922 and later four more editions appeared, including the last one in 1970. Jeffreys was a great expert and direct participant of development of the most important geophysical activities. The originality of his methodological approach to describing the material lies in that, after formulation and theoretical consideration of the problem, he writes a chapter devoted to the experimental data and facts on the theme of the comparison with analytical solutions and discussion.

Remaining on the position of Newton's and Clairaut's models, Jeffreys considers the planet as an elastic body and describes the equation of the force equilibrium from hydrostatic pressure, which appears from the outer uniform central force field, and strengths in a given point in the form

$$\rho f_i = \rho X_i + \sum_{k=1,2,3} \frac{\partial p_{ik}}{\partial x_k}, \quad (1.4)$$

where  $\rho$  is the density;  $f_i$  is the acceleration component;  $p_{ik} = p_{ki}$  is the stress component from the hydrostatic pressure;  $X_i$  is the gravity force on the unit mass from the outer force field.

Additionally, the equation of continuity (like the continuity equation in hydrodynamics) is written as the condition of equality of velocity of the mass inflow and outflow from elementary volume in the form

$$\frac{\partial \rho}{\partial t} = - \sum_i \frac{\partial}{\partial x_i} (\rho v_i), \quad (1.5)$$

where  $v_i$  is the velocity component in the direction of  $x_i$ .

Further, applying the laws of elasticity theory, he expresses elastic properties of the matter by Lamé coefficients and writes the basic equations of the strength state of the body, which links the strengths and the deformations in the point as

$$\rho \frac{\partial^2 u_i}{\partial t^2} = (\lambda + \mu) \frac{\partial \Delta}{\partial x_i} + \Delta^2 u_i, \quad (1.6)$$

where  $u_i$  is the displacement component;  $\lambda$  and  $\mu$  are the Lamé coefficients;  $\Delta$  is the component of the relative displacement;  $\Delta$  is the Laplacian operator.

One may see that Jeffreys reduced Newton's effects of gravitation to the effects of Hooke's elasticity. The author introduces a number of supplementary physical ideas related to the properties of Earth's matter, assuming that it is not perfectly elastic. With development of stresses the matter reaches its limit of resistance and passes to the stage of plastic flow with a final effect of a break in the matter continuity. This break leads to a sharp local change in the strength state, which, in turn, leads to appearance of elastic waves in the planet's body causing earthquakes. For this case Eq. (1.6) after the same corresponding transformations is converted

into the form of plane longitudinal and transversal waves, which propagate in all directions from the break place. Such is the physical basis of earthquakes which was a starting point of development of seismology as a branch of geophysics studying propagation of elastic longitudinal and transversal waves in the Earth's body. By means of seismic study, mainly by strong earthquakes and based on differences in velocity of propagation of the longitudinal and transversal waves through the shells having different elastic properties, the shell-structured body of the planet was identified.

Jeffreys analyzed the status of study in the theory of the Earth and the Moon figure following Newton's basic concepts. Namely, the planet has an inner and outer gravitational force field. The gravitational pressure is formed on the planet's surface and affects the outer space and the planet's centre. The Earth's figure is presented by an ellipsoid of rotation which is perturbed from the side of inaccuracies in the density distribution, as well as from the side of the Moon perturbations. The problem is to find the axes of the ellipsoid under action of both perturbations which occur because of a difference in the gravity field for the real Earth and the spherical body. It is accepted that the ocean's level is close to a spherical surface with deviation by a value of the first order of magnitude, and geometric oblateness of the ellipsoid is close to the value of  $e \approx 1/297$ . However, the value squares of deviation can not always be ignored because the value  $e^2$  substantially differs from the value  $e$ . The observed data cannot be compared with theoretical solutions because the formulas depending on the latitudes give precise expressions neither for the radius vector from the Earth's centre to the sea level nor for the value of the gravity force. The problem of the planet's mass density distribution finds its resolution from the condition of the hydrostatic pressure at a known velocity of rotation. The value of oblateness of the outer spheroid can be found from the observed value of the precession constant with a higher accuracy than one can find from the theory of the outer force field. A weak side of such an approach is the condition of the hydrostatic stresses, which however are very small in comparison with the pressure in the centre of the Earth. The author also notes that deviation of the outer planet's gravitational field from spherical symmetry does not satisfy the condition of the inner hydrostatic stresses. Analysis of that discrepancy makes it possible to assess errors in the inner strengths related to hydrostatics. Because of the Earth's ellipticity, the attraction of the Sun and the Moon creates the force couple applied to the centre, which forces the instantaneous axis of rotation to depict a cone around the pole of the ecliptic and to cause the precession phenomenon. The same effect initiates an analogous conclusion that was made by the author relative to the Moon's oblateness, where the observed and calculated values show much more contrast.

These are the main physical fundamentals which Jeffreys used for the analysis and theoretical consideration of the planet's figure problem and for determination of its oblateness and of semi-major axis size. The author has found that the precession constant  $H = 0.00327293 \pm 0.00000075$  and the oblateness  $1/e = 297.299 \pm 0.071$ . He assumes that the above figures could be accepted as a result which gives the hydrostatic theory. But in conclusion he says that the theory is not correct. If it is

correct, then the solid Earth would be a bench mark of the planet's surface covered by oceans. There are some other data confirming that conclusion. However, this is the only and the most precise method for determining the spheroid flattening which needs non-hydrostatic corrections to be found. An analogous conclusion was made by the author relative to the Moon's oblateness, where the observed and calculated values exhibit much more contrast.

Other review works on the irregularity of rotation and the pole motion of the Earth are the monographs of Munk and MacDonald (1964), Melchior (1972–1973), Sabadini and Vermeersten (2004). The authors analyze the state of art and geophysical causes leading to an observed incorrectness in the planet's rotation and wobbling of the poles. They draw attention of readers to practical significance of the two main effects and designate about ten causes of their initiation. Among them are seasonal variations of air masses, moving of the continents, melting and growing of the glaciers, elastic properties of planets, convective motion in the liquid core. The authors stressed that solution of any of the above geophysical tasks should satisfy the dynamical equations of motion of the rotating body and the equations, which determine a relationship between the stresses and deformations inside the body. Theoretical formulation and solution of a task should be considered on a hydrostatic basis, where the forces, inducing stresses and deformations are formed by the outer uniform force field and the deformations occur in accordance with the theory of elasticity for the elastic body model, and in frame work of rheology laws for the elastic and viscous body model. The perturbation effects are the windy forcing, the ocean currents and convective flows in the core and in the shells.

The causes of the axis rotation wobbling and pole motion are considered in detail. The authors find that the problem of precession and nutation of the axis of rotation has been discussed since old times and it does not cause any extra questions. The cause of the phenomena is explained by the Moon and the Sun's perturbation of the Earth which has an equatorial swelling and obliquity of the axis to the ecliptic. The Euler equations for a rigid body form a theoretical basis for the problem's solution. In this case the free nutation of the rigid Earth according to Euler is equal to 10 months.

Summing up the above short excursion to the problem's history we found the situation as follows. The majority of researchers dealing with dynamics of the Earth and its figure come to the unanimous conclusion that the theories based on hydrostatics do not give satisfactory results in comparison with observations. For instance, Jeffreys straightly says that the theories are incorrect. Munk and MacDonald more delicately note that a dozen of the observed effects can be called which do not satisfy the hydrostatic model. It means that dynamics of the Earth as a theory is absent. The above state of art and the conclusion gave the idea to the authors to search for a novel physical basis for dynamics of the Earth.

Newton was the founder of classic mechanics where motion is considered on the basis of his three laws, his two-body problem and the Earth's oblateness solution. Those problems were the first step in formulating and solving a very difficult problem in searching for the nature of orbital motion forces and configuration of the planet.

As to the quantitative value of the Earth's oblateness, it appears that the Newtonian value was overestimated. The Newtonian gravitation here is some natural effect of the attraction.

## 1.2 Einstein's Gravitation

In 1632 G. Galilei in his book "Dialogue about two of the main world systems—Ptolemaic and Copernican" introduced the principle of relativity which asserted equivalency of different frames of reference. In 1864 on the basis of experimental data obtained by Oersted, Ampere and Faraday, James Clerk Maxwell wrote equations of the electromagnetic field. It followed from the equations that electromagnetic waves of the field are propagated in vacuum by light velocity. The progress in farther development of the gravitation theory in the twentieth century was connected with the name of Albert Einstein.

Einstein studied mathematics and physics in Zurich's Eidgenoessische Polytechnische Schule and finished it with a diploma degree in 1900 and started the work in Berne's patent bureau. Here he began his studies in theoretical physics. On the basis of the found idea that for a non-accelerating observer the light velocity in vacuum does not depend on velocity of its source motion, Einstein concluded that the light velocity should have constant value. That fact he used for development of the special theory of relativity. In 1905 Einstein submitted in Zurich University his doctoral thesis entitled "A new determination of molecular dimensions", which was soon accepted. After that in the article "On a heuristic point of view concerning the production and transformation of light" Einstein proposed that electromagnetic radiation must consist of photons and explained photoelectric effect. That paper was reviewed by Max Plank and rejected, but soon conformed and adopted. For this work Einstein was awarded the Nobel Prize in 1921.

Einstein continued this work on general theory of relativity for several years trying to construct it as a scalar theory, but he could not find such a model. Further, in 1912 he applied to his friend and classmate in Polytechnische Schule, Marcel Grossman, to help him in construction of a mathematical model for describing his physical theory. At this time Grossman was the head of the physical-mathematical department in the Zurich Politechnische Schule. He accepted the idea of his old friend and proposed that Einstein's theory be looked at via the tensor mathematical model. Initial ideas of the tensor calculus were put out by B. Riemann and E. Christoffel in 1864, and were completed by Italian physicists G. Ricci and T. Levi-Civita in 1901.

In 1913 Einstein and Grossman prepared their first common paper under the title "The Theory of Gravitation", which described general ideas of the theory. The paper included two chapters: physical 22 pages, prepared by Einstein, and mathematical 16 pages, prepared by Grossman. The paper was published in the journal *Zeitschrift fur Matematik und Physik* в 1913 г. In 1916 Einstein published the paper "Foundation of the general theory of relativity". This is a short story of the appearance of Einstein's gravitation theory. Every 3 years since 1975, Marcel