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What is it called,
when the day rises, like today,
when everything is ruined,
everything is pillaged,
and nonetheless the air breathes,
and one has lost everything,
the city burns,
the innocents kill each other,
but the guilty are in agony,
in a corner of the day that is rising? ...
It's called dawn.

—Jean Giraudoux, *Electra*

Foreword

Series 360

The goal that the 360 series, like many degrees and thus proposing a wide-ranging horizon, has set itself may seem too ambitious. Though completeness may not be a realistic aim, the desire to propose a variety of perspectives and to especially accept direct confrontation, via a conversation which, depending on authors and subjects, could turn into a *disputation*, is at the heart of the editorial project. The principle of the series is to gather around a subject four qualified individuals and make them converse with a view of better bringing out the connections in their arguments and questioning.

This original approach is, therefore, deliberately far removed from the dominant modes of scientific "communication"—single-person monologues, usually a scientist, sociological or journalistic investigations, "beautiful books," books with a message—whose qualities and usefulness are assuredly undeniable, by proposing a dialogue around a topic involving science, never in isolation, but together with its relationship to social, economic, ideological, or political issues. In our view, the mere juxtaposition of written texts, a solution often used in publications, cannot render that which arises from the diversity of opinions, and of the various disciplines, intellectual origins, and social practices the protagonists come from. Hence the decision was taken to base the books in the 360 series on a genuine dialogue between authors, in a single time and place remindful of a theater convention. In our opinion, this genuine dialogue, which explains the oral quality of the books, though of course subsequently polished, is what makes all their wittiness and originality and justifies the risk taken by this sometimes acrobatic *modus operandi*.

This series, a recent initiative of the publishing house *La ville brûle,* started with a volume on the Big Bang Theory, the conceptual pillar of astrophysical laboratories, raised by some cosmologists to the rank of a precise theory—even an "ordinary" one, by the desire to challenge—in the last decade or so, as well as the matter of open discussions on its epistemological status that some reduce to a state of speculations (*Le big bang n'est pas une théorie comme les autres*, 2009).

The series followed with climatic changes, by confronting different perspectives from various origins—economy, climatology, and journalism—reflecting the public debates incited by this major issue (*Changement climatiques: les savoirs et les possibles*, 2010). The avatar of quantum theories that are multiple universes enabled us to make physicists, a historian and a philosopher debate (*Multivers, mondes possibles de l'astrophysique, de la philosophie et de l'imaginaire*, 2010). The burning issue of the future of nuclear power after the catastrophe of the Japanese plant in Fukushima Dai-ichi in March 2011 reproduced the configuration of the book on climate, mixing economics, technology, and journalism as evidence that this issue cannot be dealt with by confining it to only one approach (*Nucléaire: quels scéanarios pour le futur?*, 2011).

Freedom in Mathematics

The series follows with this book on mathematics. It confronts the points of view of two mathematicians, Pierre Cartier who was one of the pillars of the famous Bourbaki group and Cédric Villani, one of the most brilliant of his generation, who received the Fields Medal in 2010. Mathematician and historian of science, Jean Dhombres and philosopher of science, Gerhard Heinzmann, both specialists in mathematics, engage in a fruitful dialogue with the two mathematicians, likely to lead the reader to reflect on mathematical activity and its social consequences in mankind's history and in the modern world. Cédric Villani's popular success proves once again that common awareness has assimilated, even if in a very confused way, the major role of mathematics in the construction and efficiency of natural sciences that are at the origin of our technologies. Notwithstanding this, the idea that mathematics cannot be shared remains entrenched, and even branded as a lack of culture laid claim to by some in the media, cultural, and political establishment. Our authors explore three major directions in their dialogue. The very complex relationship between mathematics and reality is a subject of many debates and opposing viewpoints. The freedom that the construction of mathematics has given man is by enabling him to develop natural sciences, as well as that needed by mathematicians to develop their research. The responsibility with which the scientific community and governments should address the issue of the role of mathematics is in research or education policies. Happy reading!

Sylvestre Huet
Journalist for the French daily Libération
specialized in scientific information
Director of the 360 series for the publishing house *La ville brûle*

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About the Authors

Pierre Cartier Academic by training, emeritus CNRS[1] research director, research scientist at the *Institut des Hautes Etudes Scientifiques* (IHES) of Bures-sur-Yvette, former member of the Bourbaki group.

I was born in 1932 in Sedan, a town full of history. I come from two family traditions: engineers on the paternal side, and teachers on the maternal side. My mother was a very cultured, perfectly bilingual, non-conformist woman, combining love for Wagner's music as well as for Debussy and Ravel's music. In fact, a somewhat rough character to whom I owe my knowledge of the German language and my love for music. In other settings, she would have been typical of the Jewish *intelligentsia*.

Despite my provincial background, I followed the path of excellence in my studies: a secondary school in my home town, followed by the *lycée Saint-Louis* in Paris and then the *École normale superieure*, and I obtained an advanced teaching degree and a doctorate in mathematics (1958). After these *Wanderjahre*, I went to Princeton for two years; there I became acquainted with legendary figures such as Robert Oppenheimer and Andre Weil (brother of philosopher Simone Weil). Last but not least, I served in the military for a long time in the marine corps against the backdrop of the Algerian War.

Then I was a professor at the University of Strasbourg from 1961 to 1971. A major period of reconstruction: universities were experiencing tremendous expansion then, mathematics teaching needed to be completely rethought, scientific collaboration with the Germans needed to be developed after the disaster of the Second World War. In addition, the years 1950–1975 were the hey-day of the Bourbaki group, of which I was one of the pillars; there I became friends with famous people like Cartan, Schwartz, Dieudonné, Chevalley, Weil, and with younger ones.

These were also very turbulent years in the history of France: Algeria War, return of De Gaulle, construction of Europe, (worldwide) student revolutions,

[1] *Centre national de la recherche scientifique*. Translator's note.

Vietnam war. As a matter of fact, I was also well-trained as an activist: Protestant scouting (like Rocard[2] from and Jospin[3]), progressive catholic circles, Spanish anarchist refugees (after Franco's victory). I remained as a "mathematician without borders" far beyond these years, fighting against colonial wars and dictatorships both of the East and of the West. I was especially involved in Vietnam and Chile, and remain active within the framework of the cooperation agency of the French Mathematical Society (Cimpa).

I have been working in the Paris region since 1971, and I roamed between Parisian academic institutions: University Paris-Sud, *École polytechnique, École normale supérieure*. I am currently emeritus research director at the University Paris-Diderot and a visitor (for indeterminate term) at the *Institut des hautes études scientifiques* (Bures-sur-Yvette). I am not a member of the French Academy of Sciences (a voluntary choice).

My scientific interests are quite diverse (even eclectic), but centered around group theory and mathematical physics. My often cited thesis relates to algebraic geometry, but I also contributed to differential geometry, number theory, combinatorics, numerical analysis, probability and mathematical physics. I wrote a reference book on "Feynman Integrals" (in collaboration with C. DeWitt-Morette, "Functional Integration, Action and symmetries", Cambridge University Press, 2004).

I have supervised more than forty doctoral theses, and I continue to work with some of my students on multiple zeta values and the Galois theory of differential equations.

Jean Dhombres Mathematician and historian of science, emeritus CNRS research director, director of studies at the Alexandre Koyre Centre of the *École des hautes études en sciences sociales* (EHESS), specialist of the mathematics of functional equations and of their applications, of epistemology of mathematics, and of the history of scientific communities and of the spread of scholarly ideas.

I was born in Paris in 1942 in a modest and very structured household, but by force of circumstance with a limited outward orientation, and was lucky to benefit from the "thirty glorious years", and hence never having to ask myself what I should do out of necessity, but only what I wished to do in the world. Having decided to study at the *École polytechnique*, the Algerian War having ended, I chose to take a chance with mathematical research instead of graduating as an engineer, benefiting once again from a remarkable environment in Paris from 1965 onwards.

I worked in functional analysis and through a series of encounters, I joined a small international group of scientists focused on functional equations. In fact it was in Bangkok that my first book was published as a result of a talk I had given at Chulalongkorn University in 1971. Besides many articles, I am happy to have been

[2]Briefly Prime minister of France in 1988. Translator's note.

[3]Prime minister of France from 1997 to 2002. Translator's note.

able to work with Janos Aczel, the uncontested leader in this field, having published with him a book on the subject, which has now become a reference, in *Encyclopaedia of mathematics* (University of Cambridge Press, republished in 2008).

Probably as a reaction to my family background as well as to a scientific training centered on France, I wished to see the world, by learning Chinese, by participating in the establishment of experimental classes in Wu Han in China, then by becoming scientific adviser in a French embassy for three years. I became professor at the University of Nantes as early as 1972. Since for a while I was the head of its mathematics department, I was confronted with issues related to teaching and continuing education of high school teachers. Setting up a mathematics education research Institute in Nantes led me to think that getting involved in the history of mathematics would enable me to identify interesting pedagogical methods. This activity has become a major interest and led to the establishment of the Francois Viète Centre in Nantes.

I was fortunate to be elected as a director of studies at the *École des hautes études en sciences sociales* (EHESS) in 1988 and to the Chair of History of Exact Sciences, a title I chose. That same year I accepted the position of CNRS research director to run a laboratory devoted to the history of science in Paris, which now falls within the framework of the Alexandre Koyre Centre, feeling somehow the successor of this philosopher, by line of descent from René Taton and Pierre Costabel.

I am not trying to minimise epistemological influences that may have determined my choice. For the main difference between mathematical and historical practices is that the latter requires almost unending reading, whereas in the former economy and quickness are qualities. I know that I am merging two contradictory options. On the one hand, provocation, in the style of Paul Feyerabend, according to whom all is good as justification in science, and, on the other hand, the patience of the French epistemological school which consists in taking seriously all the justifications given by mathematicians, even the most dogmatic ones, while analyzing them in their cultural context, a context that is also metaphysical, social and political. I think historians still need to better account for the representation of mathematics in human societies in order to grasp how these societies function, by learning how to properly benefit from the computer revolution which gives access to all the texts, from the past to the present-day. It is indeed true that current mathematics are often useful to comprehend past mathematics, and that a celebratory tone, or mere erudite accumulation concerning the latter is of no use for us.

Emeritus since 2007, I am fortunate to be able to continue to lead a seminar at the EHESS, which in the academic year 2013 will be on the question of authority in mathematics.

Finally, let me mention the *Liber amicorum* which has been dedicated to me and which includes a list of publications (*Reminisciences*, vol. 8, Brepols, 2008), and *Une Histoire des savoirs mathématiques et de leurs pratiques culturelles. De l'émancipation à l'âge baroque à la moisson des lumières* (Hermann 2015).

Gerhard Heinzmann Professor of Philosophy at the University of Lorraine, founder of the History of science and of philosophy laboratory – Archives Henri Poincaré (UMR 71117), director of the *Maison des sciences de l'homme* of Lorraine (USR 3261).

After studying mathematics, philosophy and Greek philosophy in Heidelberg, I specialized in the philosophy of mathematics and logic. I am fascinated with crossing boundaries, with the apparent conflict between two rationalities, exemplified by the exact sciences on the one hand, and human and social sciences, on the other. I like to take a new look at ossified aspects of the obvious and the non-obvious, of the finite and the infinite, and I am aware that what was a matter of common sense 30 years ago no longer is today. In short, I am fiercely opposed to the new boundary represented by dogmatism. From this point of view, Henri Poincaré symbolizes as an idol for me: a great mathematician and a great philosopher, he trod a path between rationalism, which bases the explanation of the world on reason and empiricism, which draws knowledge from experimentation. I too also uphold this boundary as it does not separate but brings closer. It is for this very reason that I have accepted to run the *Maison des sciences de l'homme* in Lorraine.

I draw my scientific interests from a dual source, both systematic and historical: in Germany, the challenges faced in the first half of the twentieth century by formalistic, intuitionistic and logistic positions in the philosophy of mathematics have led to the development of the pragmatistic program of the Erlangen school (Paul Lorenzen, Kuno Lorenz). But I owe my historical sensitivity to Jules Vuillemin: I realised then that the pragmatistic and constructive methods used in the philosophy of mathematics to overcome the difficulties faced also stem from or at least resemble the French tradition around Poincaré, as well as intellectual circles more or less closely associated to it (Gonseth, Piaget, Cavaillès, Beth, Bernays, but also Ajdukiewicz, Brouwer and Weyl). This is especially noticeable if Ludwig Wittgenstein and Nelson Goodman are included as go-betweens for these thinkers and dialogical pragmatism. I decided to institutionalize this connection by having Goodman awarded an honorary doctorate in the framework of the Poincaré Archives in 1997.

My research focuses mostly on the use of intuitive and formal elements in mathematics and logic. No one contests that intuition is necessary for invention. However, opinions diverge as to the role intuition may be said to play in the process of understanding and justification in mathematics. Some ascribe a fundamental and irreducible role to it, others just want to exclude it. I am looking for an answer to this dilemma in a *pragmatic semantics*. It is built upon the works of Kuno Lorenz: mathematical realities cannot be conceived independently from their constructions and the latter are not independent from their language description. Instead of considering the construction and the description of objects as two different procedures of mathematical knowledge, they are regarded as two different aspects of the same process of dialogical construction from a common base of actions taking account of their goals and of the given context.