ENERGY HARVESTING WIRELESS COMMUNICATIONS

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1

Introduction

Energy harvesting (EH) is expected to have abundant applications in future wireless communication systems to power transceivers by utilizing environmental energy such as solar, thermal, wind, and kinetic energy. It becomes a promising technology that enables sensor networks, cellular networks, and wireless communications in wide rural areas.

Since renewable energy is generally clean and cheap, EH offers various benefits compared with conventional energy supplies such as batteries and fossil-fuel-based generators. For example, in cellular networks, solar panels and wind farms have been deployed to power base stations, thus lowering the expenses of energy bills, reducing the level of carbon dioxide emissions, and improving the flexibility of deployment. Besides, in wireless sensor networks, EH has been considered as a good substitute for the traditional battery, which in principle prolongs the network operation time to almost infinity. In short, EH in turn leads to a promising future for wireless networks: green and self-sustainable.

1.1 Energy Harvesting Models and Constraints

Despite many advantages, the use of EH also imposes new challenges on the design of wireless communication. Obviously, the harvested energy from solar, thermal, wind, and kinetic energy sources is not stable and might change randomly over time. Therefore, besides the randomness of the channel fading, there is another dimension of stochastic resource to be dealt with, and it brings new constraints in the optimization of EH wireless communication systems.

Wireless communication channels often fluctuate more substantially and dynamically than practical EH rates (e.g. the channel changes on the order of milliseconds, while the EH rate changes on the order of seconds or minutes), while channel fading is the main challenge faced in the design of reliable wireless communications. To illustrate this issue, we adopt a point-to-point wireless communication system, which consists of one transmitter powered by an energy harvester and one receiver with a reliable power supply, to show the phenomenon of the multi-time-scale channel/EH rate variations. In practice, the coherence time of EH processes is often much larger than that of wireless channels, as previously mentioned. Therefore, a block-based quasi-static EH model is practically valid, where the EH rate remains constant within each EH coherent block and may change from one block to another, and at the same time each EH block spans over many communication channel coherent blocks, as shown in Figure 1.1. For the

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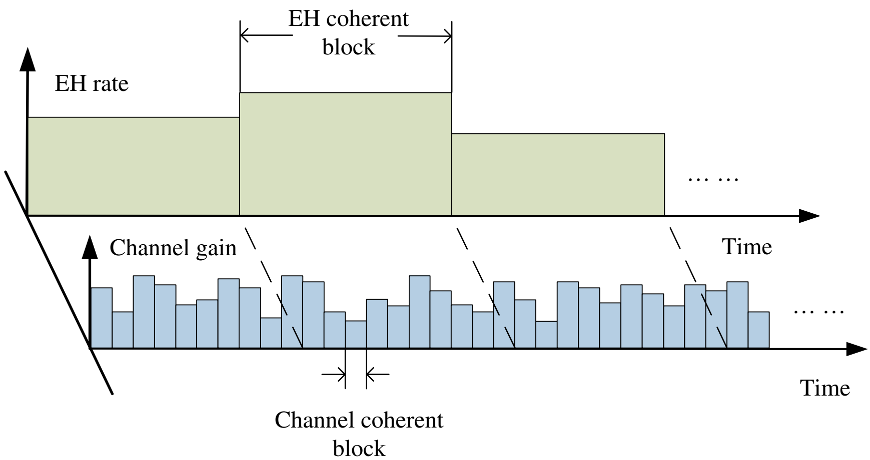

Figure 1.1 Time variations in EH process versus wireless channel.

purpose of exposition, we consider wireless data transmissions over a finite horizon of $M \leq 1$ EH blocks. Each EH block is further divided into $N \leq 1$ communication blocks each of one unit time and a constant channel gain.

Moreover, the random and intermittent characteristics of renewable energy impose a new type of EH constraint: the available energy at an EH communication node up to any time is bounded by its accumulatively harvested energy at that time. This is in contrast to conventional communication systems with stable energy sources, in which the available energy at any time is either unbounded or only limited by the remaining energy in the storage device (e.g. battery).

Mathematically, let $E_m \geq 0$ denote the EH rate in the mth EH block and $h_{n,m} \geq 0$ the channel power gain of the (n, m)th communication block (i.e. the nth communication block of the mth EH block) with $n = 1, \dots, N, m = 1, \dots, M$. Furthermore, we use $P_{n,m} \geq 0$ to denote the power consumption at the transmitter in the (n, m)th communication block. Unless otherwise stated, we consider that $P_{n,m}$ represents the transmit power at the transmitter and ignore the power consumption by circuit, signal processing, etc. Assuming an ideal energy storage device (i.e. with infinite capacity and no energy leakage) employed at the transmitter, we have the *EH constraints* on the scheduled power consumptions $\{P_{n,m}\}$; that is, the energy accumulatively consumed up to any communication block (n, m), i.e. $\sum_{j=1}^{m-1}\left(\sum_{i=1}^{N} P_{i,j}\right) + \sum_{i=1}^{n} P_{i,m}$, should be no larger than the energy accumulatively harvested by then, i.e. $N\sum_{j=1}^{m-1} E_j + nE_m$. In other words, we have the EH constraints as

$$\sum_{j=1}^{m-1}\sum_{i=1}^{N} P_{i,j} + \sum_{i=1}^{n} P_{i,m} \leq N\sum_{j=1}^{m-1} E_j + nE_m, \quad n = 1, \dots, N, m = 1, \dots, M. \tag{1.1}$$

Due to both the new EH constraints and the multi-time-scale channel/EH rate variations, it is a challenging problem to jointly optimize the communication scheduling and energy management in EH-based wireless communications.

Finally, the availabilities of the channel state information (CSI) $\{h(n, m)\}$ and the energy state information (ESI) $\{E(m)\}$ at the transmitter, respectively, can significantly affect the performance of EH communication systems. Among all different assumptions about the channel state information at the transmitter (CSIT) and energy state information at the transmitter (ESIT), there are four cases of primary interest in this book, listed below:

(1) *Case 1*: Noncausal CSIT and ESIT. At the beginning of the transmission, the transmitter perfectly knows the past, current, and future CSI and ESI. This case approximates the practical scenario when the transmitter can accurately predict the future CSI (e.g. slowly varying channels in low-mobility applications) and the future ESI (e.g. based on historical data in a periodically varying energy environment). The optimal solution in this case provides a performance upper bound for all other CSIT/ESIT availability cases.
(2) *Case 2*: Causal CSIT and ESIT. At the beginning of each EH/communication block, the transmitter knows the past and current CSI/ESI, as well as the statistical information (e.g. distributions) of future CSI/ESI. In general, the solution of this case achieves the lowest utility among the first three cases considered herein.
(3) *Case 3*: Causal CSIT and noncausal ESIT. This is a hybrid model based on cases 1 and 2, in which all ESI is perfectly known at the beginning of the transmission while only the past and current CSI is known.
(4) *Case 4*: No CSIT and noncausal/causal ESIT. During the transmission, the transmitter does not have any CSI and only has statistical information on the CSI. The noncausal or causal ESIT is defined as that in Case 1 or 2 above. Note that in all the above cases, we assume that at each communication block, the receiver perfectly knows the CSI in that block.

1.2 Structure of the Book

Based on the previous section, it is observed that EH brings a new dimension to the wireless communication problems, in the form of intermittency and randomness of the available energy, as well as the possibility of the energy cooperations among the transmission nodes in wireless networks. In this book, we summarize the progresses taken in the past few years in this new research field. This book is divided into three parts:

(1) In part I, we focus on the optimal transmission design for EH wireless communication systems. In particular, Chapter 2 addresses the optimal power allocation problems for the point-to-point EH channels to maximize the system throughput or minimize the average outage probability and also considers the effects of imperfect circuits and limited battery storage. Chapter 3 examines the power allocation for various multi-node wireless channels powered by energy harvesters, including the multiple-access channels (MACs), relay channels, and large relay networks. Chapter 4 studies the cross-layer design for EH communications, considering some upper layer issues such as transmission delay and traffic variations over time.
(2) In part II, we focus on the design and optimization of some EH networks. Chapter 5 considers the *ad hoc* networks, where there is no central control of the whole network, and studies the opportunistic access control schemes and the corresponding throughput scaling behavior. Chapter 6 considers a standard cellular network with multiple base stations powered by energy harvesters and studies the energy and

communication cooperations among them. Chapter 7 considers several new issues in the next-generation cellular networks and studies EH-based hyper-cellular networks with control and traffic separation and proactive content caching and push for better utilization of the renewable energy on small-cell base stations.

(3) Part III includes three appendices about the basic tools widely used in this book.

Part I

Energy Harvesting Wireless Transmission

2

Power Allocation for Point-to-Point Energy Harvesting Channels

To start with, in this chapter we consider the simplest point-to-point energy harvesting (EH) channel with a single EH-powered wireless transmitter communicating with a single wireless receiver. As shown in Figure 2.1, the transmitter can use an energy harvester to harvest the renewable energy from the environment and then store the harvested energy in a rechargeable storage device, to be used for sending information to the receiver. On the other hand, the receiver can either be powered by fixed energy supply (such as battery or grid) or EH, which will be specified later. The aim of this chapter is to characterize the fundamental limits of the communication performance of the point-to-point EH channel via designing the power allocation at the wireless transmitter. As this channel can be viewed as a building block of more general multiuser EH channels and EH-powered wireless networks, the results obtained in this chapter will provide key design insights for general EH networks.

In the point-to-point EH channel, the introduction of renewable energy imposes new design challenges on the power allocation at the wireless transmitter. In particular, as the renewable energy arrival rates or EH rates are generally random and intermittent, a new type of EH constraints is introduced, i.e. the available energy at the EH wireless transmitter up to any time is bounded by its accumulatively harvested energy by that time. This is in sharp contrast to conventional wireless communication systems with fixed energy sources, in which the available energy at any time is either unbounded or only limited by the remaining energy in the storage device (e.g. battery). In addition, wireless communication channels and practical EH rates often fluctuate at different time scales, while channel fading is the main challenge faced in the design of reliable wireless communications. Due to both the new EH constraints and the multi-time scale channel/EH rate variations, it is a challenging problem to design the power allocation at the EH wireless transmitter for communication performance optimization [1–3].

In this chapter, we first build a general utility optimization framework to reveal the key design principles of the point-to-point EH channel by considering the scenario with the receiver powered by a fixed energy supply and an ideal transmitter with an infinite energy storage and perfect circuits. Then, we apply this framework to solve two specific problems including the throughput maximization and the outage probability minimization, which are commonly adopted in the wireless literature as the communication performance metrics. Next, we extend the power allocation strategies to more practical setups by considering the limited energy storage and imperfect circuits at the wireless transmitter. In addition, we discuss the power allocation when both the transmitter and the receiver are EH powered.

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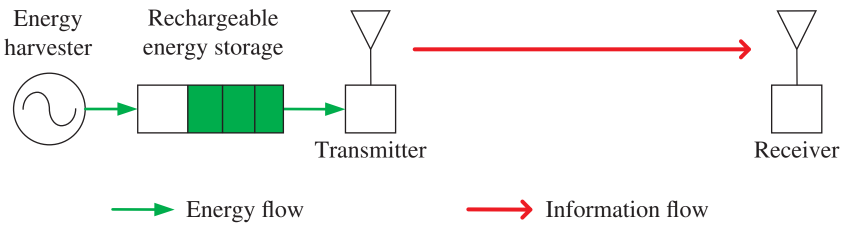

Figure 2.1 A point-to-point EH channel with an EH transmitter using the harvested power to send information to a receiver.

2.1 A General Utility Optimization Framework for Point-to-Point EH Channels

In the following sections, we focus on the case when the receiver has a fixed energy supply, unless otherwise stated. We consider block-based quasi-static models for the EH process at the wireless transmitter as well as the wireless communication channel from the transmitter to the receiver, where both the EH rate and the wireless channel remain constant over each block and may change from one block to another. Since the coherence time of the EH process is often much larger than that of the wireless channel as previously mentioned, we consider that each EH coherent block spans over many communication channel coherent blocks, as shown in Figure 2.1. For the purpose of exposition, we consider the wireless data transmission over a finite horizon of length T (in, e.g. seconds) that consists of $M \geq 1$ EH blocks. Each EH block is further divided into $N \geq 1$ communication blocks each of one unit time and a constant channel gain.

Let $E_m \geq 0$ denote the EH rate in the mth EH block and $h_{n,m} \geq 0$ the channel power gain of the (n, m)th communication block (i.e. the nth communication block of the mth EH block) with $n = 1, \dots, N, m = 1, \dots, M$. Furthermore, we use $P_{n,m} \geq 0$ to denote the power consumption at the transmitter in the (n, m)th communication block. Unless otherwise stated, we consider that $P_{n,m}$ represents the transmit power at the transmitter and ignore the power consumption by circuit, signal processing, etc. Assuming an ideal energy storage device (i.e. with infinite capacity and no energy leakage) employed at the transmitter, we have the *EH constraints* on the scheduled power consumptions $\{P_{n,m}\}$; that is, the energy accumulatively consumed up to any communication block (n, m), i.e. $\sum_{j=1}^{m-1} \left(\sum_{i=1}^{N} P_{i,j} \right) + \sum_{i=1}^{n} P_{i,m}$, should be no larger than the energy accumulatively harvested by then, i.e. $N \sum_{j=1}^{m-1} E_j + nE_m$ [4, 5]. In other words, we have the EH constraints as

$$\sum_{j=1}^{m-1} \sum_{i=1}^{N} P_{i,j} + \sum_{i=1}^{n} P_{i,m} \leq N \sum_{j=1}^{m-1} E_j + nE_m, \quad n = 1, \dots, N, \; m = 1, \dots, M. \tag{2.1}$$

Over the (n, m)th communication block, we denote the input–output relationship from the transmitter to the receiver in the point-to-point wireless channel as

$$y_{n,m} = \sqrt{P_{n,m} h_{n,m}} x_{n,m} + v_{n,m}, \quad n = 1, \dots, N, \; m = 1, \dots, M, \tag{2.2}$$

where $y_{n,m}$ is the channel output or the received signal at the receiver, $x_{n,m}$ is the corresponding channel input or the transmitted signal by the transmitter with zero mean and unit average power, and $v_{n,m}$ is the independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian (CSCG) noise at the receiver with zero mean and unit variance. For each communication block, the achievable data rate (in bps Hz^{-1}) corresponds to the instantaneous mutual information of the channel, assuming the optimal Gaussian codebook for the transmitted signals, which is given by

$$I_{n,m}(h_{n,m}, P_{n,m}) = \log_2(1 + h_{n,m}P_{n,m}). \tag{2.3}$$

To characterize the communication quality of service (QoS) measured at the receiver, we define a general utility function $U_{n,m}(P_{n,m})$ for the (n, m)th block, which is dependent on the instantaneous mutual information $I_{n,m}(h_{n,m}, P_{n,m})$ at that block. As a result, the general utility maximization problem over the M EH blocks could be formulated as

$$\max_{\{P_{n,m}\geq 0\}} \sum_{m=1}^{M} \sum_{n=1}^{N} U_{n,m}(P_{n,m}) \tag{2.4}$$

$$\text{s.t.} \sum_{j=1}^{m-1} \sum_{i=1}^{N} P_{i,j} + \sum_{i=1}^{n} P_{i,m} \leq N \sum_{j=1}^{m-1} E_j + nE_m,$$
$$n = 1, \dots, N, \ m = 1, \dots, M. \tag{2.5}$$

In practice, the utility function $U_{n,m}(P_{n,m})$ can be defined more explicitly as throughput [4–10], non-outage probability [11, 12], or other performance metrics such as end-to-end distortion in an EH-based estimation system [13], which could be either deterministic or statistical average based on the availabilities of the channel state information (CSI), i.e. $\{h_{n,m}\}$, and the energy state information (ESI), i.e. $\{E_m\}$, at the transmitter, namely, CSIT and ESIT, respectively. In the following sections, we will particularly focus on the throughput maximization and outage probability minimization by considering the throughput and the non-outage probability as the utility $U_{n,m}(P_{n,m})$. Moreover, among all different assumptions about the channel state information at the transmitter (CSIT) and energy state information at the transmitter (ESIT), there are four cases of our primary interest including:

- *Case 1*: Noncausal CSIT and ESIT.
- *Case 2*: Causal CSIT and ESIT.
- *Case 3*: Causal CSIT and noncausal ESIT.
- *Case 4*: No CSIT and noncausal/causal ESIT.

Note that in all the above cases, we consider the case that at each communication block, the receiver perfectly knows the CSI in that block.

2.2 Throughput Maximization for Gaussian Channel with EH Transmitter

First, we consider the throughput maximization problem in the EH channel by considering the instantaneous mutual information $I_{n,m}(h_{n,m}, P_{n,m})$ in (2.3) as the utility function. Specifically, we consider the Gaussian channel case with the channel power gains remaining constant over the whole time horizon. As a result, we have $h_{n,m} = h, n = 1, \dots, N, m = 1, \dots, M$. In this case, the utility function is expressed as

$$U_{n,m}(P_{n,m}) = I_{n,m}(h, P_{n,m}) = \log_2(1 + hP_{n,m}) \tag{2.6}$$

in bps Hz^{-1} with normalized noise power. Furthermore, with the Gaussian channel, we have the number of communication blocks in each EH block as $N = 1$ and define $P_m \triangleq P_{1,m}$ without loss of generality. Accordingly, the general utility maximization problem in (2.4) can be expressed as the following throughput maximization problem:

$$\max_{\{P_m \geq 0\}} \sum_{m=1}^{M} \log_2(1 + hP_m) \tag{2.7}$$

$$\text{s.t.} \sum_{j=1}^{m} P_j \leq \sum_{j=1}^{m} E_j, \quad m = 1, \dots, M. \tag{2.8}$$

The optimal power allocation solution to the problem (2.7) depends on the availability of the ESIT. In the following, we consider two cases with noncausal and causal ESIT, respectively.

2.2.1 The Case with Noncausal ESIT

When the transmitter noncausally knows the current and future ESI, we refer to the solution to problem (2.7) as an *offline* optimization. In the following, we first obtain the optimal solution structure to problem (2.7) and then discuss its implementation in practice.

2.2.1.1 Staircase Power Allocation to Problem (2.7)

It is easy to show that problem (2.7) is a convex optimization problem, since the objective function is concave and the EH constraints are linear. As a result, this problem can be efficiently solved via standard convex optimization techniques [14]. Furthermore, since problem (2.7) satisfies Slater's condition, strong duality holds between (2.7) and its dual problem. In this case, we can apply the Karush–Kuhn–Tucker (KKT) conditions to reveal its optimal solution.

Let $\lambda_m \geq 0$ denote the Lagrange multiplier associated with the mth constraint in (2.8) and $\mu_m \geq 0$ denote the Lagrange multiplier associated with the power constraint $P_m \geq 0, m = 1, \dots, M$. The Lagrangian associated with problem (2.7) is

$$\mathcal{L}_{2.2}(\{P_m\}, \{\lambda_m\}, \{\mu_m\}) = \sum_{m=1}^{M} \log_2(1 + hP_m) - \sum_{m=1}^{M} \lambda_m \left(\sum_{j=1}^{m} P_j - \sum_{j=1}^{m} E_j \right) + \sum_{m=1}^{M} \mu_m P_m. \tag{2.9}$$

Then the necessary and sufficient conditions for $\{P_m^\star\}$, $\{\lambda_m^\star\}$, and $\{\mu_m^\star\}$ to be the primal and dual optimal solutions to problem (2.7) are given by the following KKT conditions:

$$0 \leq P_m^\star, \tag{2.10}$$

$$0 \geq \sum_{j=1}^{m} P_j^\star - \sum_{j=1}^{m} E_j, \tag{2.11}$$

$$0 \le \lambda_m^\star, \tag{2.12}$$

$$0 \le \mu_m^\star, \tag{2.13}$$

$$0 = \lambda_m^\star \left(\sum_{j=1}^{m} P_j^\star - \sum_{j=1}^{m} E_j \right), \tag{2.14}$$

$$0 = \mu_m^\star P_m^\star, \tag{2.15}$$

$$0 = \frac{\partial \mathcal{L}_{2.2}(\{P_m^\star\}, \{\lambda_m^\star\}, \{\mu_m^\star\})}{\partial P_m^\star}, \tag{2.16}$$

for all $m = 1, \dots, M$. From (2.10), (2.13), (2.15), and (2.16), we obtain the optimal power allocation as

$$P_m^\star = \left[\nu_m - \frac{1}{h} \right]^+, \quad m = 1, \dots, M, \tag{2.17}$$

where $[x]^+ \triangleq \max(x, 0)$, $\nu_m \triangleq \left(\ln 2 \sum_{j=m}^{M} \lambda_j^\star \right)^{-1} \ge 0$, and the $\lambda_j^\star$'s satisfy the above KKT conditions.

For the ease of description, we define an EH block $t \in \{1, \dots, M\}$ as a *transition block* if the transmit power changes after EH block t, i.e. $P_t^\star \ne P_{t+1}^\star$. We define the last EH block M also as a transition block (say, by defining $P_{M+1}^\star$ to be infinity); hence there is at least one transition block. We collect all transition blocks as the set $\mathcal{S} = \{t_1, \dots, t_{|\mathcal{S}|}\}$, where $t_i < t_j$ for $i < j$ and $t_{|\mathcal{S}|} = M$. Then, we have the following structural properties for the optimal power allocation $\{P_m^\star\}$, as shown in Figure 2.2.

Proposition 2.1 *The optimal solution $\{P_m^\star\}$ in (2.17) to problem (2.7) satisfies the following properties:*

- *The optimal transmit power monotonically increases over time, i.e. $P_1^\star \le P_2^\star \le \cdots \le P_M^\star$. We say that the optimal solution performs staircase power allocation over blocks, since the transmit power $\{P_m^\star\}$ in (2.17) is a staircase-like function.*
- *If EH block t is a transition block, then the battery storage is empty after this block, or equivalently, the accumulative energy consumed up to EH block t equals that harvested up to then, i.e. (2.11) holds with equality for $t \in \mathcal{S}$.*

Proof: Since $\lambda_j^\star \ge 0$, it follows that $\nu_1 \le \nu_2 \le \cdots \le \nu_M$. Accordingly, based on (2.17), we have $P_1^\star \le P_2^\star \le \cdots \le P_M^\star$. Therefore, the first property is verified.

Suppose that the transmit power changes after EH block t, and thus we have $\nu_t \ne \nu_{t+1}$. Since by definition $\nu_t = \left(\ln 2 \sum_{j=t}^{M} \lambda_j^\star \right)^{-1}$, we get $\lambda_t^\star \ne 0$. From (2.13), we get $\lambda_t^\star > 0$. It then follows from the complementary slackness condition (2.14) that (2.11) holds with equality for EH block t. This proves the second property. □

2.2.1.2 Efficient Algorithm to Solve Problem (12.7)

Based on the structural properties of the optimal solution, we develop an efficient algorithm to implement the staircase power allocation to optimally solve problem (2.7). Such an algorithm was proposed initially in [4] and in [6] in parallel.

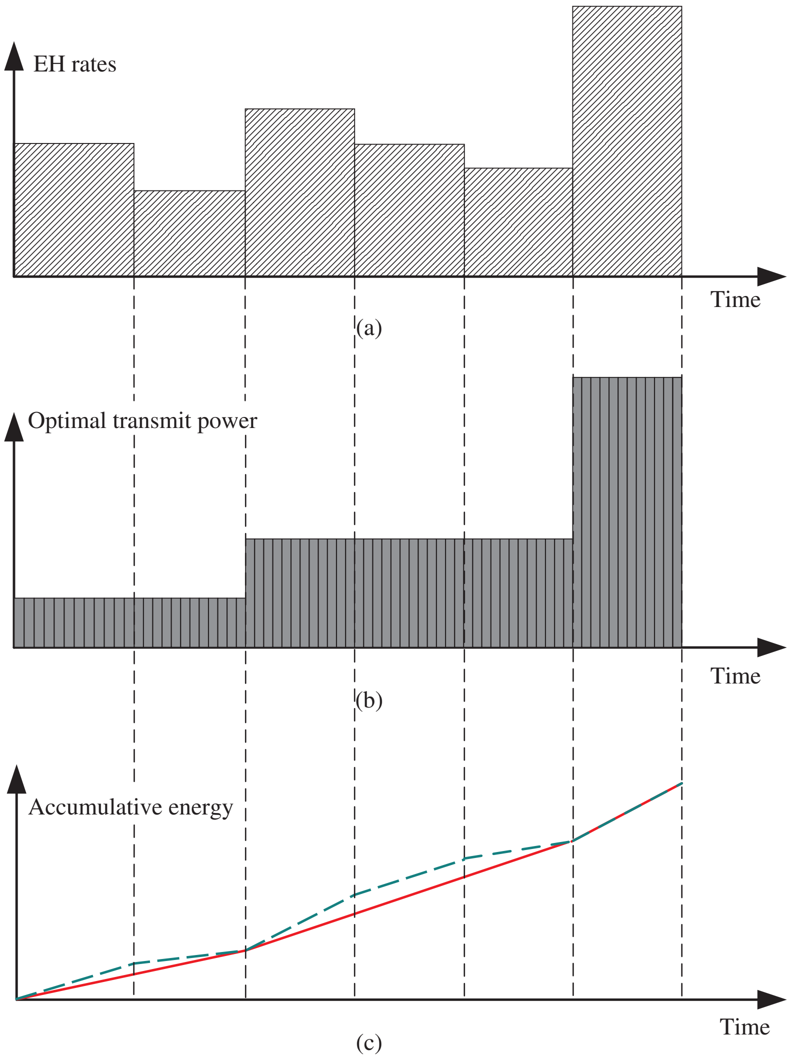

Figure 2.2 Staircase structure of the optimal transmit power allocation. (a) The EH rates over time. (b) The optimal staircase transmit power over time. (c) The accumulatively harvested energy (upper curve) and accumulatively consumed energy (lower curve) over time.

Some definitions are in order. For convenience, let $t_0 = 0$. We define the ith *block interval* $\tilde{S}_i$, where $i = 1, \dots, |S|$, as the EH blocks between the ith and the $(i+1)$th transition block, i.e. $\tilde{S}_i \triangleq \{t_{i-1}+1, \dots, t_i\}$. Thus, $\cup_i \tilde{S}_i = \{1, \dots, M\}$ and $\tilde{S}_i \cap \tilde{S}_j = \phi$ for $i \neq j$. Let $S^\star = \{t_1^\star, \dots, t_{|S^\star|}^\star\}$ denote the optimal set of transition blocks corresponding to an optimal power allocation.

Then the optimal staircase power allocation is performed as follows. From the second property in Proposition 2.1, all the harvested energy available in the ith block interval (i.e. $\sum_{m=t_{i-1}^\star+1}^{t_i^\star} E_m$) needs to be used during the ith block interval. Moreover, the optimal transmit power should remain constant over this block interval. In other words, the optimal power allocation at the ith block interval is given as

$$P_m^\star = \frac{\sum_{m=t_{i-1}^\star+1}^{t_i^\star} E_m}{t_i^\star - t_{i-1}^\star}, \quad m = t_{i-1}^\star + 1, \dots, t_i^\star. \tag{2.18}$$

As a result, the original optimization problem (2.7) can thus be reduced to a search for the optimal transition block set $\mathcal{S}^\star$ that has a size from 1 to at most M:

$$\max_{1\le|\mathcal{S}|\le M} \max_{\mathcal{S}} \sum_{i=1}^{|\mathcal{S}|} (t_i - t_{i-1}) \log\left(1 + h\frac{\sum_{m=t_{i-1}+1}^{t_i} E_m}{t_i - t_{i-1}}\right) \tag{2.19}$$

subject to the power allocation $\left\{P_m = \frac{\sum_{m=t_{i-1}+1}^{t_i} E_m}{t_i - t_{i-1}}\right\}$, satisfying the EH constraints in (2.8). A brute force search based on (2.19) is of a high computational complexity. Nevertheless, it turns out that it is optimal to simply employ a forward-search procedure, starting with the search of the optimal $t_1^\star$, then of the optimal $t_2^\star$, and so on until the last optimal transition block $t_{|\mathcal{S}^\star|}^\star$ equals M, at which point the optimal size $|\mathcal{S}^\star|$ is also obtained.

The first optimal $t_1^\star$ can be found in Lemma 2.1 given below; by induction, the search of the subsequent optimal transition blocks will follow similarly. Lemma 2.1 requires the following *feasible-search procedure*:

(1) Initialize $\mathcal{S}_1$ as an empty set.
(2) For $t_1 = 1, \dots, M$, obtain the optimal power allocation from block 1 to block t_1 as

$$P_{t_1} = \frac{\sum_{m=1}^{t_1} E_m}{t_1}, \quad t_1 = 1, \dots, M. \tag{2.20}$$

(3) Admit t_1 in the set $\mathcal{S}_1$ if the corresponding optimal power allocation satisfies the EH constraints in (2.8).

Note that the set $\mathcal{S}_1$ is non-empty, as it contains at least the element $t_1 = 1$. Moreover, the set $\mathcal{S}_1$ includes all possible candidates for the optimal $t_1^\star$.

Lemma 2.1 *Let $\mathcal{S}_1$ be the feasible set of t_1 obtained by the feasible-search procedure. Then the optimal transition block is given by the largest element in $\mathcal{S}_1$, which corresponds to the element with the smallest value of $\left\{\frac{\sum_{m=1}^{t_1} E_m}{t_1}\right\}$, i.e.*

$$t_1^\star = \max_{t_1\in\mathcal{S}_1} t_1 = \arg\min_{1\le t_1\le M} \frac{\sum_{m=1}^{t_1} E_m}{t_1}. \tag{2.21}$$

Proof: If $|\mathcal{S}_1| = 1$, then the only t_1 must be optimal. Henceforth, assume $|\mathcal{S}_1| \ge 2$. Consider two blocks $t', t'' \in \mathcal{S}_1$, where $t' < t''$. Denote their respective optimal transmit power obtained from (2.20) as $P_{t'} = \frac{\sum_{m=1}^{t'} E_m}{t'}$ and $P_{t''} = \frac{\sum_{m=1}^{t''} E_m}{t''}$. Then $P_{t'} \ge P_{t''}$. Otherwise if $P_{t'} < P_{t''}$, then more power is allocated for each time block $k = 1, \dots, t'$, with the transmit power $P_{t''}$ used, as compared with the case with $P_{t'}$ used. But since the transmit power $P_{t'}$ has used all available power at block t' due to second property in Proposition 2.1, the transmit power $P_{t''}$ is infeasible and thus cannot be optimal.

We now show that t' cannot be the optimal $t_1^\star$ by contradiction. Suppose that $t_1^\star = t'$, i.e. the transmit power $P_{t'}$ is used from block 1 to block t'. The transmit power must then subsequently decrease at some block $t' < k \leq t''$; otherwise the sum power allocated from block 1 to block t'' will be more than the sum power allocated with the (constant) transmit power $P_{t''}$, which violates the sum power constraint. But from the first property in Proposition 2.1, the optimal transmit power is non-decreasing. Thus, $t_1^\star \neq t'$ by contradiction. By induction, all elements in $\mathcal{S}_1$, except for the largest one, are suboptimal. The only candidate left, namely, the largest element, must then be optimal.

Based on the above discussion, $t_1^\star$ should correspond to the element with smallest value of $\left\{P_{t_1} = \frac{\sum_{m=1}^{t_1} E_m}{t_1}\right\}$ from (2.20). As a result, it must follow that $t_1^\star = \arg\min_{1\leq t_1 \leq M} \frac{\sum_{m=1}^{t_1} E_m}{t_1}$, and this lemma is proved. □

We now propose Algorithm 2.1 to find the optimal transition blocks for solving problem (2.19) or equivalently problem (2.7), for which the optimality is shown in Proposition 2.2. Briefly, Algorithm 2.1 computes $t_1^\star, t_2^\star, \dots, t_{|\mathcal{S}^\star|}^\star$ in each of the iteration until $t_{|\mathcal{S}^\star|}^\star = M$. Given that $t_1^\star, \dots, t_{i-1}^\star$ is found, $t_i^\star$ is obtained based on (2.21) in Lemma 2.1.

Proposition 2.2 *Algorithm 2.1 obtains the optimal $\mathcal{S}^\star$ that solves the optimization problem in (2.19) or equivalently problem (2.7).*

Proof: The ith iteration of Algorithm 2.1 finds the optimal $t_i^\star$ based on (2.21) in Lemma 2.1. Next, from the second property in Proposition 2.1, in the first block interval, all the power available would be used. Since no power is available for subsequent block intervals given $t_1^\star$, the power allocation for subsequent block intervals can be optimized independent of the actual power allocated in the first block interval. The throughput maximization problem from block $t_1^\star + 1$ onward can be solved similarly as before (after removing time blocks $1, \dots, t_1^\star$). Thus, we determine $t_2^\star$ based on (2.21), similarly for $t_3^\star$ and so on, as reflected in the iteration of Algorithm 2.1. The iteration ends when the optimal transition block equals M, which is the largest possible value as stated in the optimization problem in (2.19). □

Algorithm 2.1: Finding the optimal transition blocks.

Input: $t_0^\star = 0$
Output: $t_i^\star$
for $i = 1, 2, \dots, M$ **do**
  $P_{t_i} \leftarrow \frac{\sum_{j=t_{i-1}^\star+1}^{m} E_j}{m - t_{i-1}^\star}$
  $t_i \leftarrow t_{i-1}^\star + 1$
  $t_i^\star \leftarrow \arg\min_{t_{i-1}^\star+1 \leq t_i \leq M} P_{t_i}$
  if $t_i^\star - M$ **then**
    Exit the algorithm
  end if
end for

To better illustrate the optimal power allocation structure at the EH transmitter, Figure 2.2c shows the accumulatively harvested energy (the upper curve) and accumulatively consumed energy (the lower curve) over time. Here, both the upper and the lower curves increase monotonically over time, and their slopes represent the EH rate and the optimal transmit power, respectively. First, the accumulatively consumed energy curve is observed to always lie below the accumulatively harvested energy curve, which is in order to satisfy the EH constraints. Next, the slope of the accumulatively consumed energy curve is observed to monotonically increase over time, which is expected as in the first property of Proposition 2.1. Furthermore, it is observed that when the slope of the accumulatively harvested energy curve changes, it touches the accumulatively harvested energy curve at that time instant. This is consistent with the second property of Proposition 2.1, showing that the harvested energy must be used up at that time instant. This also implies that the first line segment of the accumulatively consumed energy curve should be the line connecting the original and the corner point of the accumulatively harvested energy curve and with the minimum slope, for which the idea is used to develop Algorithm 2.1 to obtain the optimal staircase power allocation solution.

2.2.2 The Case with Causal ESIT

Next, we consider the other case with only causal ESIT, i.e. at each EH block m, the transmitter only knows the knowledge of past and present E_j's, $j = 1, \dots, m$, but is not aware of future E_j's, $j = m + 1, \dots, M$. In this case, the transmitter cannot solve problem (2.7) via the offline optimization approach with convex optimization techniques due to the unawareness of E_j's, $j = m + 1, \dots, M$. Alternatively, we need to use an *online* optimization for problem (2.7) in the following.

2.2.2.1 Dynamic Programming

Dynamic programming is known as the optimal online approach to solve problem (2.7), provided that the harvested energy E_m's follow a stochastic process with certain distributions, and the transmitter knows such distribution information. In this case, the transmitter aims to maximize the expected throughput over the finite horizon of M EH blocks, i.e. $\mathbb{E}\left(\sum_{m=1}^{M} \log_2(1 + hP_m)\right)$, subject to the EH constraints in (2.8). The policy can be optimized offline and implemented in real time via a look-up table that is stored at the transmitter.

At each EH block m, we denote the state of the system as the harvested energy E_m and the energy storage level, denoted by B_m, at that block. Here, B_m's are given as

$$B_m = \sum_{i=1}^{m-1} (E_i - P_i), \quad m = 1, \dots, M, \tag{2.22}$$

where $B_1 = 0$. Note that with B_m at hand, the EH constraints in (2.8) can be re-expressed as

$$P_m \leq B_m + E_m, \quad m = 1, \dots, M. \tag{2.23}$$

Then we have the following proposition.

Proposition 2.3 *Given E_1 and B_1, the optimal value achieved by maximizing $\mathbb{E}\left(\sum_{m=1}^{M}\log_2(1+hP_m)\right)$ subject to the EH constraints in (2.8) is given by $J_1(E_1, B_1)$, which can be computed recursively based on the following Bellman equations, starting from $J_M(E_M, B_M)$, $J_{M-1}(E_{M-1}, B_{M-1})$, and so on until $J_1(E_1, B_1)$:*

$$J_M(E_M, B_M) = \max_{P_M \geq 0} \ \log_2(1+hP_M) \quad \text{s.t. } P_M \leq B_M + E_M, \tag{2.24}$$

$$J_m(E_m, B_m) = \max_{P_m \geq 0} \ \log_2(1+hP_m) + \bar{J}_{m+1}(B_m - P_m) \quad \text{s.t. } P_m \leq B_m + E_m, \tag{2.25}$$

for $m = 1, \ldots, M-1$, where

$$\bar{J}_{m+1}(B_m - P_m) = \mathbb{E}_{E_{m+1}}(J_{m+1}(E_{m+1}, B_m - P_m)), \tag{2.26}$$

where $\mathbb{E}_{E_{m+1}}(\cdot)$ denotes the expectation over E_{m+1}. An optimal policy is accordingly given by $\pi^\star = \{P_m^{\mathrm{DP}}(E_m, B_m)\}$, where $P_m^{\mathrm{DP}}(E_m, B_m)$ is the optimal solutions to problem (2.24) for $m = M$ and (2.25) for $m = 1, \ldots, M-1$.

Proof: The proof follows directly by applying Bellman equations [15] and thus is omitted here for brevity. □

For problem (2.24), the optimal solution is trivial, i.e. $P_M^{\mathrm{DP}}(E_M, B_M) = B_M + E_M$, which means that the transmitter uses all the available energy for transmission in the last EH block M. We can interpret the maximization in (2.25) as a tradeoff between the present and future rewards. This is because the throughput $\log_2(1+hP_m)$ represents the present reward, while $\bar{J}_{m+1}(B_m - P_m)$, commonly known as the value function, is the expected future throughput accumulated from block $m+1$ until block M.

Next, we present structural properties of the maximum throughput $J_m(E_m, B_m)$'s and the corresponding optimal policy $\pi^\star$ in Propositions 2.4 and 2.5. The two propositions are proved based on the concavity of the throughput function $\log_2(1+P_m)$, for which the details can be found in [4].

Proposition 2.4 *We have the following properties for $J_m(E_m, B_m)$'s:*

(1) $J_m(E_m, B_m)$'s in (2.24) and (2.25) are concave in B_m for $m = 1, \ldots, M$.
(2) $\bar{J}_{m+1}(B_m - P_m)$ in (2.26) is concave in B_m for $m = 1, \ldots, M$.

Proposition 2.5 *Given E_m, the optimal power allocation $P_m^{\mathrm{DP}}(E_m, B_m)$ that solves (2.24) and (2.25) is non-decreasing in B_m, where $m = 1, \ldots, M$.*

The structural properties in Propositions 2.4 and 2.5 simplify the numerical computation of the optimal power allocation solution in Proposition 2.3. In particular, from (2.24), we get the optimal solution for block M as $P_M^{\mathrm{DP}}(E_M, B_M) = B_M + E_M$. Now, consider the problem of finding the optimal $P_m^{\mathrm{DP}}(E_m, B_m)$ to obtain $J_m(E_m, B_m)$, $m = 1, \ldots, M-1$. As $\bar{J}_{m+1}(B_m - P_m)$ in (2.26) is concave as shown in Proposition 2.4, the objective function in (2.25) is concave. Thus, problem (2.25) is a convex optimization

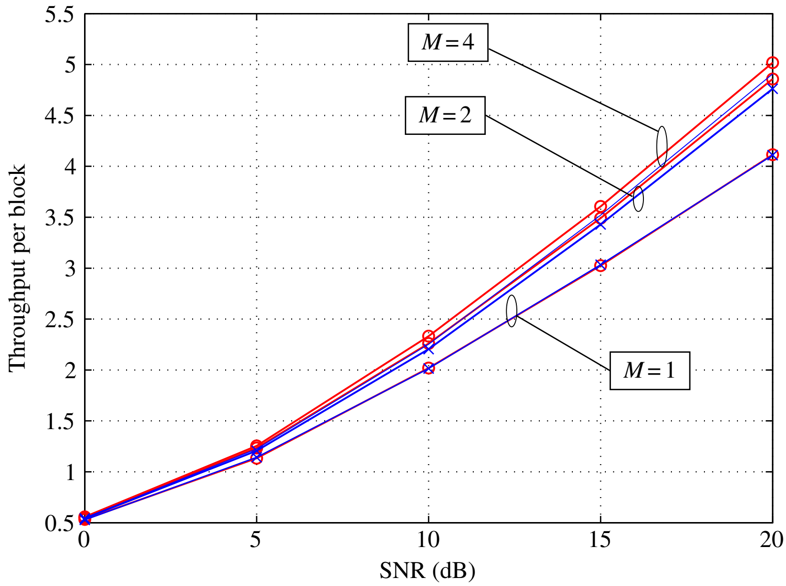

Figure 2.3 Optimal throughput with noncausal ESI (light gray with "o" markers) or causal ESI (dark gray with "×" markers) at the transmitter for $M = 1, 2, 4$.

problem and has a unique solution, which can thus be easily solved using numerical techniques such as a bisection search [14].

Figure 2.3 compares the throughput per EH block, i.e. the sum throughput divided by the number of EH blocks M, with noncausal ESI versus that with causal ESI. In this simulation, the harvested energy E_m's are i.i.d. over EH blocks m. We assume the harvested energy E_m takes a value in $\{0, 0.5, 1\}$ with equal probability. It is observed that for $M = 1$, the throughput in both cases with noncausal ESI and causal ESI is the same, because any ESI cannot be exploited for future EH blocks. However, in both cases the throughput per block increases as M increases. The increment is more substantial when noncausal ESI is available, intuitively because the ESI can then be much better exploited. The incremental improvement as M increases is significant when M is small but becomes less significant when M is large. The throughput with either noncausal ESI or causal ESI does not differ significantly, possibly because the ESI that can be further exploited from noncausal ESI is limited in our i.i.d. scenario.

2.3 Throughput Maximization for Fading Channel with EH Transmitter

Now, we move to the case with fading channels, where the transmitter aims to maximize the throughput over the finite horizon with M EH blocks each consisting of N communication blocks. In this case, the utility function is given as

$$U_{n,m}(P_{n,m}) = \log_2(1 + h_{n,m}P_{n,m}) \quad (2.27)$$

in bps Hz^{-1} with normalized noise power for the (n, m)th block. Problem (2.4) can then be reformulated as

$$\max_{\{P_{n,m}\geq 0\}} \sum_{m=1}^{M}\sum_{n=1}^{N} \log_2(1 + h_{n,m}P_{n,m}) \quad \text{s.t. } (2.5). \tag{2.28}$$

As for the CSIT and ESIT, we particularly focus on Case 1 with noncausal CSIT and ESIT and Case 2 with causal CSIT and ESIT. We will provide some discussions on the other two cases, i.e. Case 3 with causal CSIT and noncausal ESIT, as well as Case 4 with no CSIT and noncausal/causal ESIT.

2.3.1 The Case with Noncausal CSIT and ESIT

First, we consider the case with noncausal CSIT and ESIT. In this case, we adopt an offline optimization to solve problem (2.28). Since the objective function is concave and the constraints are all linear, problem (2.28) is a convex optimization problem. Furthermore, as problem (2.28) satisfies Slater's condition, strong duality holds between problem (2.28) and its dual problem. In this case, we use the KKT condition to obtain the optimal solution.

2.3.1.1 Water-Filling Power Allocation

Before proceeding to consider the general case where the EH constraint (2.5) is imposed for all $m = 1, \dots, M$ and $n = 1, \dots, N$, we impose the constraint (2.5) only for the last communication block, i.e. only for $(n, m) = (N, M)$. This then corresponds to the conventional problem of maximizing the sum throughput with a *sum* energy constraint of $P_{\max} = N \sum_{m=1}^{M} E_m$:

$$\mathcal{T}^{\mathrm{WF}}(\{h_{n,m}\}_{(1,1)}^{(N,M)}, P_{\max}) = \max_{\{P_{n,m}\geq 0\}} \sum_{m=1}^{M}\sum_{n=1}^{N} \log_2(1 + h_{n,m}P_{n,m}) \tag{2.29}$$

$$\text{s.t.} \sum_{m=1}^{M}\sum_{n=1}^{N} P_{n,m} \leq P_{\max}. \tag{2.30}$$

Since less constraints are imposed, the maximum throughput in (2.29) is no smaller than that of problem (2.28). It is well known that the optimal solution to problem (2.29) is given by (see, e.g. [14, 16])

$$P_{n,m}^{\mathrm{WF}} = \left[\nu - \frac{1}{h_{n,m}}\right]^{+}. \tag{2.31}$$

This optimal solution is implemented by the *water-filling power allocation algorithm*, where the *water-level* (WL) $\nu \geq 0$ is chosen such that (2.30) holds with equality by using the optimal power allocation in (2.31). For completeness, an implementation of the *water-filling power allocation algorithm* is given in Algorithm 2.2.