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Jochen Kühn

Optimal Risk-Return Trade-Offs of Commercial Banks

and the Suitability of Profitability Measures for Loan Portfolios

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Jochen Kühn

Volkenbachstraße 4
79798 Jestetten
Germany

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Contents

1

Introduction

1.1 Problem Statement and Research Question

Active loan portfolio management is becoming more and more important. In the year 2004, European banks sold credits worth EUR 249 billion. Big deals were made by the German banks Hypo Real Estate (EUR 3.6 billion) and Dresdner Bank (EUR 1.2 billion). In addition, credit exchanges were established which made loans more liquid. For example, in October 2004 the German "Deutsche Kredit-Börse" was established, which focuses on trading loans assigned to medium-size businesses.

It is empirically shown that active loan portfolio management can be very profitable.[1] However, a precondition to benefit from active loan portfolio management is having knowledge about valuating loan portfolios. Shareholders can steadily benefit from such transactions only if banks valuate loan portfolios correctly. This is this dissertation's motivation for dealing with profitability measures for loan portfolios.

Nowadays, banks measure the profitability of loan portfolios primarily by calculating the return on risk adjusted capital (RORAC). Here return is the expected profit after refinancing and operational costs. Risk adjusted capital, more frequently called economic capital, is the amount of equity which must be held to guarantee a certain given solvency level of the bank.

However, calculating this ratio is not sufficient when valuating loan portfolios. The calculation of economic capital implies that the bank

[1] See Cebenoyan and Strahan (2004).

already knows which solvency level is optimal. It also presumes that the optimal solvency level is independent of the risk-return profile of the loan portfolio. But this need not be true.

Think about a bank without operational costs that can decide between two risk-return profiles of its loan portfolio (see Table 1.1). The first profile A is characterized by an expected return of 6% and low risk, the second profile B has an expected return of 6.2% and a comparatively higher risk. The bank has equity of 100 and needs to fulfill the condition of requiring less economic capital than the equity it has. The calculation of economic capital is based on a certain given solvency level. Given the risk-return profile of its loan portfolio, the bank can adjust the required economic capital by changing the loan portfolio volume.

First, assume that the bank has a high given solvency level leading to an interest rate on debt of 5%. With the less risky profile A, the bank can hold a portfolio with a volume of 1500. Having the riskier profile B, the bank can only hold a portfolio with a volume of 1200 due to the economic capital restriction. Choosing profile A, this leads to an expected profit after refinancing costs of $6.00\% \cdot 1500 - 5.00\% \cdot (1500 - 100) = 20$ and to a RORAC of 20%. Choosing profile B, an expected profit after refinancing costs of $6.20\% \cdot 1200 - 5.00\% \cdot (1200 - 100) = 19.4$ and a RORAC of 19.4% result. It could be concluded that profile A is preferable.

But now assume that it is actually optimal for the bank to have a lower solvency level with profile B, while at the same time the given solvency level is optimal for profile A. Furthermore, assume that the bank with the lower solvency level has to pay a higher interest rate on debt of 5.1%, but it can also have a higher portfolio volume of 1450, choosing profile B. The expected profit after refinancing costs thus amounts $6.20\% \cdot 1450 - 5.10\% \cdot (1450 - 100) = 21.05$ and the RORAC is 21.05% when profile B and the lower solvency level are chosen. So it is optimal for the bank to decide in favor of profile B.

This example shows that valuating loan portfolios based on RORAC with a given solvency level can be misleading for the optimal solvency level depending on the risk-return profile of the loan portfolio. So it would be a significant improvement to have a sound profitability measure that valuates loan portfolios directly on the basis of its risk-return profile.

Table 1.1. RORAC with a given solvency level

profile	expected return	risk	solvency	refinancing	volume	RORAC
A	6.00%	low	high	5.00%	1500	20.00%
B	6.20%	high	high	5.00%	1200	19.40%
B	6.20%	high	low	5.10%	1450	21.05%

In the literature, two profitability measures are explicitly proposed for optimizing loan portfolios based on its risk-return profile: the Sharpe ratio, which relates the expected excess return over the risk-free rate to the standard deviation of the portfolio return, and the reward-to-VaR ratio, which relates the expected excess return over the risk-free rate to a certain quantile of the excess return.[2] Furthermore, in the context of asymmetric returns, which are typical for banks, reward-to-shortfall ratios are popular. Reward-to-shortfall ratios relate the expected excess return over the risk-free rate to lower partial moments or root lower partial moments of the return.

However, the above reward-to-risk ratios are founded on capital market models, assuming that banks should optimize their loan portfolios the same way as individual capital market investors do. But this need not hold true. Rather, banks should optimize their loan portfolio, targeting at the maximization of the shareholder value. Here banks do not only need to consider the market risk premium, but also additional costs that risk-taking provoke. Thus, it is questionable whether profitability measures, derived from capital market models, reflect optimal risk-return trade-offs of banks.

The dissertation addresses this problem. Its research question is whether reward-to-risk ratios derived from capital market models are suitable for loan portfolios. The approach of the dissertation is to endogenously derive optimal risk-return trade-offs of commercial banks and to compare them to the risk-return trade-offs of the reward-to-risk ratios derived from capital market models. This gives measures such as the Sharpe ratio and the reward-to-VaR ratio a more adequate foundation for valuating loan portfolios.

[2] See Altman and Saunders (1998), pp. 1728-1740, Campbell, Huisman, and Koedijk (2001), and Alexander and Baptista (2003).

1.2 Outline of the Dissertation

The dissertation is divided into nine chapters (see Figure 1.1):

After the introduction, the following two chapters present fundamentals for the derivation of reward-to-risk ratios in the fourth chapter. The second chapter, *Risk Measures*, gives a survey of risk measures quantifying risk in the reward-to-risk ratios. The third chapter, *Asset Pricing*, derives the stochastic discount factor model and provides fundamentals for understanding capital market models, upon which the reward-to-risk ratios are based. Furthermore, the presumptions are pointed out as being necessary for a firm to maximize the market price of equity as a substitute for the shareholders' individual valuations of equity. This justifies the use of the market price of equity as the target figure for a bank.

In the fourth chapter, *Reward-to-Risk Ratios*, several reward-to-risk ratios measuring the profitability of portfolios are derived from capital market models. The starting point is the derivation of the Sharpe ratio from the stochastic discount factor model discussed in the previous chapter.

The next three chapters focus on the derivation of optimal risk-return trade-offs of commercial banks. The derived optimal risk-return trade-offs are based on the effects of risk-taking on shareholder value. They are discussed in the fifth chapter, *Effects of Risk-Taking in Commercial Banks*. The sixth chapter, *Risk-Return Trade-Offs for Commercial Banks*, is the central part of the dissertation. It develops models for endogenously deriving optimal risk-return trade-offs of commercial banks. The models assume that banks only have uninsured debtholders. The seventh chapter, *Deposits and the Risk-Return Trade-Off*, extends the models by taking into account the fact that banks are usually also financed through deposits, which are insured and have a senior credit ranking in most countries.

In the eighth chapter, *Profitability Measures for Loan Portfolios*, the endogenously derived optimal risk-return trade-offs of commercial banks are compared to the risk-return trade-offs of profitability measures derived from capital market models in order to assess their suitability for loan portfolios.

Finally, the ninth chapter, *Conclusion*, provides a summary and points out the implications and limitations of the dissertation.

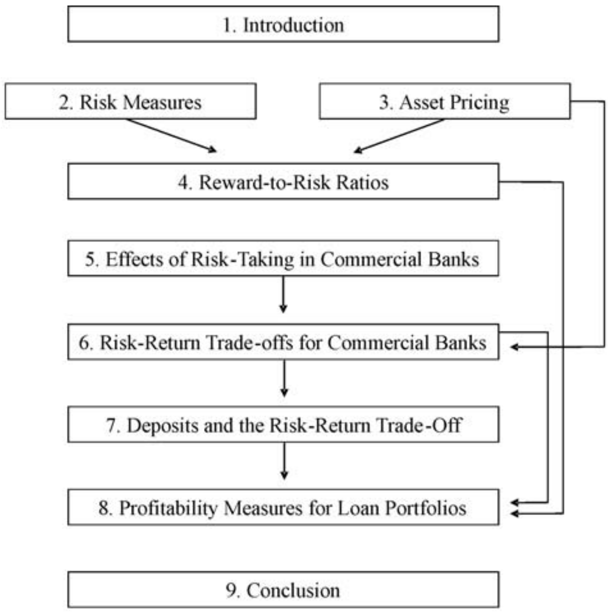

Figure 1.1. Outline of the dissertation.

2

Risk Measures

This chapter discusses risk measures upon which the assessed reward-to-risk ratios are based. It starts with a short presentation of the two most common definitions of risk.

2.1 Defining Risk

Risk is defined in at least two ways, each having a different focus.

The best known definition of risk stems from Frank Knight distinguishing between measurable and unmeasurable uncertainty and defining risk as the former one.[1] This definition emphasizes that risk is closely related to uncertainty and tasks dealing with risk usually require uncertainty to be quantified. Using this definition, uncertainty can be risk, even if it cannot cause harm.

A more intuitive understanding of risk leads to defining risk as a hazard that emanates from uncertainty and is caused by harmful deviations from expectations.[2] According to this definition, risk incorporates two basic elements: uncertainty and harm that can arise, although it is not expected.[3] Using this definition, something might be risky, although the uncertainty is not measurable.

Both definitions have advantages. The first definition is useful, e.g. for describing model risk since it helps to distinguish between uncertainty that is captured by the model and residual uncertainty. The

[1] See Knight (1921), p. 233.

[2] See Crowe and Horn (1967) and Athearn (1971).

[3] See Holton (2004).

second definition is particularly useful when it is important to distinguish between deviations from expectations leading to harm and those leading to benefits. Since this is important when dealing with risk-return trade-offs, the following discussion of risk measures is based on the second definition.

2.2 Variance and Standard Deviation

The classical risk measure is the variance Var and the square root of it called standard deviation σ. Given a density function f of a continuous square-integrable random variable r, they are defined as

$$Var(r) = \sigma^2(r) = \int_{-\infty}^{\infty} (r - \mathbb{E}(r))^2 f(r)dr \tag{2.1}$$

and

$$\sigma(r) = (\int_{-\infty}^{\infty} (r - \mathbb{E}(r))^2 f(r)dr)^{\frac{1}{2}}. \tag{2.2}$$

Here $\mathbb{E}$ denotes the expectation operator.

When r is the return of a portfolio, the variance of the return is the expected value of the squared deviation of the return from the expected return. It is well known for being used in the seminal work of Markowitz (1952). This work provides a quantitative framework for measuring portfolio risk and for deriving efficient frontiers, which characterize portfolios that maximize the expected return for a given variance of the return or minimize the variance of the return for a given expected return. The advantage of using the variance as a risk measure is that aggregating risk is quite simple using a covariance matrix. The main disadvantage of this mean-variance approach is that it presumes either normally distributed returns or a quadratic utility function of investors. This is discussed in the following.

The mean-variance approach assumes investors who invest an amount I in a portfolio with a payoff X based on the first two statistic moments of the portfolio return r, the expected return $\mathbb{E}(r)$ and the variance of the return $Var(r)$, with

$$r = \frac{X}{I} - 1. \tag{2.3}$$

This is plausible for any risk averse investor when there is no other income if the return distribution is completely specified by the mean and variance, since the variance is a sufficient risk measure in this case. A distribution fulfilling this property is the normal distribution.

Without this distribution assumption, the mean-variance approach leads to correct results if it is assured that investors themselves only care about the first two statistic moments. According to the axioms of von Neumann and Morgenstern (1944), this holds for investors with quadratic utility functions U of the form

$$U(X) = \alpha X^2 + X \text{ with } \alpha \in (-1, 0). \tag{2.4}$$

In this case, the expected utility is

$$\mathbb{E}(U(X)) = \int_{-\infty}^{\infty} U(X) f(X) dX = \alpha \int_{-\infty}^{\infty} X^2 f(X) dX + \int_{-\infty}^{\infty} X f(X) dX, \tag{2.5}$$

and with

$$Var(X) = \int_{-\infty}^{\infty} (X - \mathbb{E}(X))^2 f(X) dX = \int_{-\infty}^{\infty} X^2 f(X) dX - E^2(X), \tag{2.6}$$

the expected utility can be expressed as

$$\mathbb{E}(U(X)) = \alpha(Var(X) + \mathbb{E}(X)) + \mathbb{E}(X). \tag{2.7}$$

Since α is negative, the expected utility of investors increases in the expected payoff and decreases in the variance of the payoff. Thus, the expected utility increases in the expected return and decreases in the variance of the return, too.

However, the assumed utility function has the undesirable property that it implies an absolute risk aversion

$$ARA(X) = -\frac{U''(X)}{U'(X)} = -\frac{2\alpha}{2\alpha X + 1} \tag{2.8}$$

and a relative risk aversion of investors

$$RRA(X) = -X \frac{U''(X)}{U'(X)} = -\frac{2\alpha X}{2\alpha X + 1} \tag{2.9}$$