566 LECTURE NOTES IN ECONOMICS AND MATHEMATICAL SYSTEMS

Michael Genser

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Economic and Empirical Issues

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To my wife Astrid

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Bergen, August 2005 Michael Genser

Contents

List of Symbols

$\mathbf{1}_{\mathcal{A}}$	Indicator function: Assumes 1 if the condition $\mathcal{A}$ is true, else 0
α	Fraction of the value of the bankrupt firm V_B which is lost in bankruptcy
$\alpha(V_B)$	Bankruptcy loss function
α_1, α_1	Parameters of the bankruptcy loss function
A_1, A_2	Constants of the general solution of the ordinary differential equation of claim F if the state variable follows an arithmetic Brownian motion
$\bar{A}_1, \bar{A}_2$	Constants of the general solution of the ordinary differential equation of claim F if the state variable follows a geometric Brownian motion
$\mathcal{A}_i$	All sets of $\mathcal{N}$ with i integers
B_{C_j,T_j}	Market value of a government bond issue with maturity T_j and continuous coupon C_j after taxation
BC	Current value of the bankruptcy costs
BL	Current value of the bankruptcy losses
$\mathcal{B}_n$	Set of all joint events $\mathcal{B}_n$ with $i = \{1, \ldots, n\}$
C_j	Continuous coupon of bond j
C_j^*	Optimal continuous coupon of bond j
CF	Regular payments of a claim F
δ_L	Payout function to all claimants of the levered firm value
δ_U	Payout function to all claimants of the unlevered firm value
d_n	Vector of observed debt prices in period n

D_{C_j,T_j}	Market value of the j-th corporate debt issue with maturity T_j and continuous coupon C_j after taxation
$D^{+}_{C_j,T_j}$	Market value of the j-th corporate debt issue with maturity T_j and continuous coupon C_j after taxation as long as the firm remains solvent
$D^{-}_{C_j,T_j}$	Market value of the j-th corporate debt issue with maturity T_j and continuous coupon C_j after taxation in the case of bankruptcy
$D^{E}_{C_j,T_j}$	Market value of the j-th corporate debt issue with maturity T_j and continuous coupon C_j to equity owners
$DO(T, \eta, \eta_B)$	Down and out option value of EBIT with starting EBIT η, barrier η_B, and maturity T
$dz^{\mathcal{P}}$, $dz^{\mathcal{Q}}$	$\mathcal{P}$-, $\mathcal{Q}$-Brownian motion
ε	Fractional loss of tax recovery in Tax Regime 3
$\varepsilon_{d,n}$	Vector of debt price observation errors in period n
$\varepsilon_{e,n}$	Equity price observation error in period n
ε_n	Vector of observation errors in period n: $\varepsilon_n = (\varepsilon_{e,n}, \varepsilon'_{d,n})'$
$\eta, \bar{\eta}$	Earnings before interest and taxes (EBIT) following an arithmetic or geometric Brownian motion
$\hat{\eta}_n$	A-posteriori update of EBIT in the Kalman filter recursion in period n
$\bar{\eta}_{n\|n-1}$	A-priori update of EBIT in the Kalman filter recursion in period n given the estimate of period $n-1$
$\eta_B(T_j), \bar{\eta}_B(T_j)$	EBIT at which the firm declares bankruptcy in the time interval $]T_{j-1}, T_j]$
$\eta^*_B, \bar{\eta}^*_B$	EBIT at which the equity owners optimally declare bankruptcy
$\eta_{\max,t}$	Maximum EBIT earned from investments
e_n	Observed equity price in period n
E	Market value of equity to investors after taxation
E^{+}	Market value of equity to investors after taxation as long as the firm remains solvent
E^{-}	Market value of equity to investors after taxation in the case of bankruptcy

$E_{n\|n-1}$	Expectation operator of period n given information in period $n-1$
$E_t^{\mathcal{Q}}$, $E_t^{\mathcal{P}}$	Expectation operator under the probability measure $\mathcal{Q}$ or $\mathcal{P}$ as of time t
$\mathcal{E}$	Set of all even integers in $\mathcal{N}$
$f(e_n\|e_{n-1})$	Conditional density of e_n given e_{n-1}
$f(\eta_t)$	Payout function of an EBIT claim
$F(\eta_t, T)$	Market value of a claim contingent on the firm's EBIT with maturity T
$g(\eta_t, \eta_B(t))$	Function that transforms the time varying barrier $\eta_B(t)$ with respect to η_t into a constant barrier with respect to $g(\cdot)$
G	Market value of tax payments collected by the government
G^+	Market value of tax payments collected by the government as long as the firm remains solvent
G^-	Market value of tax payments collected by the government in the case of bankruptcy
G_η	Gradient of the model security prices with respect to the state variable η
h	Spacing parameter in the Stirling approximation
$\hat{H}$	Information matrix of the Kalman filter
$\hat{H}_{E,n}$	Asymptotic variance of the equity price estimation error
$\hat{H}_{D,n}$	Asymptotic covariance matrix of the bond price estimation errors
I	Identity matrix
I_t	Invested capital as of time t
k	Proportional debt issuing costs
k_1, k_2	Characteristic roots of the general solution of the ordinary differential equation of security prices in the ABM-Corporate Securities Framework
$\bar{k}_1$, $\bar{k}_2$	Characteristic roots of the general solution to security prices of the ordinary differential equation of security prices in the GBM-Corporate Securities Framework
$K(\cdot)$	Debt issuing cost function
K_n	Kalman gain matrix
λ	EBIT spacing parameter in the trinomial lattice approach

$L(\cdot)$	(Log) likelihood function
μ, $\bar{\mu}$	Drift function of EBIT under the risk-neutral measure $\mathcal{Q}$ if EBIT follows an arithmetic or geometric Brownian motion
μ_η, $\bar{\mu}_\eta$	Drift function of EBIT under the physical measure $\mathcal{P}$ if EBIT follows an arithmetic or geometric Brownian motion
μ_{BL}	Drift function of bankruptcy losses BL
μ_L, μ_U	Drift function of the levered firm value V_L and the unlevered firm value V_U
μ_{TAD}	Drift function of tax advantage to debt claim TAD
M_{t_i}	Process of the running maximum of the process X in the subperiod $]t_{i-1}, t_i]$
ν	Drift of the stochastic process X
$\tilde{\nu}$	Drift of stochastic process X with the bankruptcy claim as numéraire
$n(\mu, \sigma^2)$	Univariate normal density function with expected value μ and standard deviation σ
$n_n(\mu, \Sigma)$	n-variate normal density function with a vector of expected values μ and the covariance matrix Σ
$N(\mu, \sigma^2)$	Univariate cumulative normal distribution function with expected value μ and standard deviation σ
$N_n(\mu, \Sigma)$	n-variate cumulative normal distribution function with a vector of expected values μ and the covariance matrix Σ
$\mathcal{N}$	Set of all integers $\{0, \ldots, n\}$
Ω	Time covariance matrix of a Brownian motion
$\phi(t_0, T, \eta_{t_0}, \eta_T, \eta_B(t))$	Joint probability of the process η starting at η_{t_0} of not hitting the barrier function $\eta_B(T)$ in the time interval $]t_0, T]$ and ending at $d\eta = \eta_T$
$\Phi(t_0, T, \eta_{t_0}, \eta_B(t))$	Probability of the process η starting at η_{t_0} of hitting the barrier function $\eta_B(T)$ in the time interval $]t_0, T]$
$\psi(t_0, s, \eta_{t_0}, \eta_B(t))$	First passage time density of the process η starting at η_{t_0} of hitting the barrier function $\eta_B(T)$ in the time interval $]t_0, T]$
p_j	Reflection counter
$P(\mathcal{A})$	Probability of the event $\mathcal{A}$

P_j	Principal of bond j
$p_B(t_0, T, \eta_{t_0}, \eta_B(t))$	Present value of a security paying one account unit if the process η starting at η_{t_0} reaches $\eta_B(T)$ for the first time in the time interval $]t_0, T]$
$\mathcal{P}$	Physical probability measure
PR	Par coupon of a government bond
$PV(\cdot)$	Present value of some payment function
PY	Par yield of a corporate bond
PYS	Par yield spread as the difference between corporate and government par yields
$\mathcal{Q}$	Risk-neutral martingale measure
r	Continuously compounded risk-free interest rate
R	Covariance matrix of the observation errors ε
$ROI_{\max}$	Maximum return on investment
σ_η, $\bar{\sigma}_\eta$	Volatility function of EBIT if EBIT follows an arithmetic or geometric Brownian motion
$\hat{\sigma}_E$	Estimated equity volatility
$\hat{\sigma}_V$	Estimated firm value volatility
σ_{IV}	Implied Black/Scholes volatility
σ_U	Volatility function of the unlevered firm value
$\bar{\Sigma}_\eta(n\|n-1)$	A-priori update of the variance of the state variable η in period n given the information in period $n-1$
$\hat{\Sigma}_\eta(n)$	A-posteriori update of the variance of the state variable η in period n
$\bar{\Sigma}_Y(n\|n-1)$	A-priori update of the variance of the observation estimates Y in period n given the information in period $n-1$
$\bar{\Sigma}_{Y\eta}(n\|n-1)$	A-priori update of the covariance of the observation estimates Y with the state variable η in period n given the information in period $n-1$
s	Point in time
S_j	Start date of the j-th corporate bond issue
τ	Bankruptcy stopping time
t_0	Point in time (today)
T	Maturity of a security
TAD	Tax advantage to debt
T_O	Option maturity
τ^c	Corporate tax rate
τ^d	Investor's personal tax rate on coupon payments
τ^e	Investor's personal tax rate on dividends

τ^{eff}	Equity investor's effective tax rate on dividends equalling $(1-\tau^e)(1-\tau^c)-1$
θ	Risk premium of the EBIT-process to change the physical measure $\mathcal{P}$ into the risk-neutral measure $\mathcal{Q}$
Θ	Parameter vector
$\hat{\Theta}_{ML}$	Maximum-likelihood estimate of the parameter vector Θ
$\hat{\Theta}_{SAE}$	Estimate of the parameter vector Θ using the sum of absolute errors as an objective function
$\hat{\Theta}_{SSQE}$	Estimate of the parameter vector Θ using the sum of squared errors as an objective function
$\mathcal{U}$	Set of all uneven integers in $\mathcal{N}$
v_n	Vector of security price estimation errors in period n
V	Value of the firm before taxation
V^+	Value of the solvent firm before taxation
V^-	Value of the bankrupt firm at the bankruptcy level η_B before taxation
$V_B(T_j)$	Value of the bankrupt firm before taxation in the subperiod $]T_{j-1}, T_j]$
V_{C_j,T_j}	Value of debt with maturity T_j and a continuous coupon C_j before taxation
$V^+_{C_j,T_j}$	Value of debt with maturity T_j and a continuous coupon C_j before taxation as long as the firm is solvent
$V^-_{C_j,T_j}$	Value of debt with maturity T_j and a continuous coupon C_j before taxation in the case of bankruptcy
V_E	Value of equity before taxation
V_E^+	Value of equity before taxation as long as the firm is solvent
V_E^-	Value of equity before taxation in the case of bankruptcy
w_j	Fraction of the total recovery value $V_B - \alpha(V_B)$ received by investors in debt issue j before taxation
w_E	Fraction of the total recovery value $V_B - \alpha(V_B)$ received by equity investors before taxation
X	Stochastic variable hitting a barrier pattern y_{t_i}

y_n	Vector of security price observations in period n: $y_n = (e_n, d'_n)'$
y_s	Continuous corporate bond yield in period s
y_{t_i}	Upper barrier of the process X in the subperiod $]t_{i-1}, t_i]$
Y	Vector of observation functions in the Kalman filter
$\bar{Y}_{n\|n-1}$	A-priori estimate of observed security prices in the Kalman filter in period n
YS_s	Continuous yield spread between corporate and government bonds in period s

1

Introduction

In the last few years, a refined pricing of corporate securities has come into focus of academics and practitioners. As empirical research showed, traditional asset pricing models could not price corporate securities sufficiently well. Time series properties of quoted securities were difficult to replicate.

In the search for more advanced models that capture the empirical findings, researchers followed two approaches. The first stream of research fitted the time series properties of corporate securities directly. We refer to this class of models as being of reduced form. Security prices are assumed to follow more advanced stochastic models, in particular models with e.g. non-constant volatility.[1] All studies of this type do not consider the economics of the issuing companies but simply assume a stochastic behavior of the security or its state variables. In contrast, a second, economic literature developed by studying the firm. We call these kinds of models structural because the limited liability of equity holders is modeled explicitly as a function of firm value.

One problem of the reduced form approach is its difficulty of interpretation in an economic sense. Being technically advanced, reduced form models often lack an intuitive economic model and especially disguise the economic assumptions. If security pricing is the only purpose of the exercise, we might not need an economic model. However, if we want to understand price movements, a serious link with the underlying economics appears important.

The credit risk literature even adopted this particular terminology to categorize its models.[2] Whereas reduced form models take each corpo-

[1] See e.g. Stein and Stein (1991) for a stochastic volatility model and Heston and Nandi (2000) on GARCH option pricing.

[2] See e.g. Ammann (2002).

rate security separately and model a firm's default by a Poisson event[3], structural credit risk models concentrate on a model of the firm value. Bankruptcy occurs when either the firm value falls for the first time to a sufficiently low level so that equity holders are not willing to support the firm for a longer period of time, or when some contractual condition forces the firm into bankruptcy. The setup of structural models allows extensions into refined decision making and the use of game theoretic arguments.

Structural credit risk models were pioneered by the seminal papers of Black and Scholes (1973) and Merton (1974). They assume that the firm value follows a geometric Brownian motion. The firm has one finite maturity zero coupon bond outstanding that the firm will repay if the terminal firm value exceeds the debt notional at maturity. Otherwise the firm defaults on its debt. Black and Cox (1976) extend this setup by allowing bankruptcy before debt maturity when the firm value touches a bankruptcy barrier for the first time.

Further extension of the basic setup introduced optimal future capital structure changes. These dynamic capital structure models were analyzed e.g. in Fischer, Heinkel and Zechner (1989a) and Fischer, Heinkel and Zechner (1989b). In both papers the capital structure of the firm is modeled endogenously in a continuous-time setting assuming equity holders to optimize the value of their claim. They do not concentrate on credit risk and bankruptcy but use an argument from corporate finance in order to explain empirically observed leverage ratios and call premia of callable corporate bond issues. The idea of equity holders maximizing the value of their claim when levering the firm or issuing callable debt is developed further by Leland (1994) and Leland and Toft (1996). They focus on the valuation of corporate debt and the sensitivity of debt value to certain model parameters, extending the Fischer et al. (1989a) framework, and derive a firm value level at which equity owners endogenously trigger bankruptcy, thus linking the dynamic capital structure with credit risk models.

However, dynamic capital structure models of the first generation caused confusion. The model dynamics is driven by a stochastic process of the unlevered firm value which can be interpreted as the value of a fully equity financed firm. All other values of interest such as the levered firm value, debt values, leverage ratios, etc. are derived in an optimal budgeting decision with respect to the process of this unlevered firm value. In such a setup, however, both the levered and unlevered

[3] See e.g. Jarrow and Turnbull (1995), Jarrow, Lando and Turnbull (1997), Duffie and Singleton (1999), Collin-Dufresne, Goldstein and Hugonnier (2004).

value of the firm exist at the same time. The pricing of these securities is only arbitrage-free under certain conditions which are usually not clearly stated because they are not obvious if one models the firm value directly.[4]

One reason for this confusion about levered or unlevered firm values in dynamic capital structure models is due to the lack of a precise definition of firm value. One could think of the market value of assets as a natural candidate. However, the market value of the assets is different to the value generated by these assets. The introduction of corporate and personal taxes, bankruptcy cost, and debt blurs the models further and misleads interpretation. The other reason is that the firm value is modeled directly whereas payments to holders of corporate securities are defined in terms of cash flows. Being unclear about which claimant receives which cash flow, it can happen that the total amount of cash flows paid out to claimants does not sum to the firm's available funds. Taking the investment policy as given and unchanged, the mismatch leads to inconsistencies in the models.[5]

More recent approaches of dynamic capital structure models, e.g. Goldstein et al. (2001), Christensen et al. (2000), and Dangl and Zechner (2004), assume a stochastic process for an income measure that is unaffected by the capital structure decision. Earnings before interest and taxes (EBIT) or free cash flow (FCF) are natural candidates. Both income measures describe the earnings or cash flows of a firm from which the interests of **all** financial claimants, such as stockholders, bondholders, and the government, must be honored. The total firm value – i.e. the value of all claims – can be determined by discounting the income measure by an appropriate discount rate. One of the most important advantages of EBIT-based capital structure models is therefore the thinking in discounted cash flows that generate value. It forces a split of the EBIT into different claims, thus easing the interpretation

[4] Some authors like Goldstein, Ju and Leland (2001, p. 485) try hard to convince the reader that it is reasonable to model unlevered firm values by the argument that unlevered firms exist. They quote Microsoft as an example. However, this argument is void since nobody can prevent firms from issuing debt. So even the price histories of the stocks of these firms already account for the *potential* of a capital budgeting decision optimizing leverage in the future. On the other hand, if Microsoft does not issue debt although there is some tax advantage to do so, there must be a reason if they opt out. Again, none of the models can explain this kind of behavior. See Christensen, Flor, Lando and Miltersen (2000, p. 4f.) for a review of this argument.

[5] A prominent example of inconsistency is the numerical example in Goldstein et al. (2001), where EBIT does not match the sum of coupon, dividend and tax payments. Such a case is not covered in their model.