

ERINGMATHEMATICAENGINEERINGMATHEMATI

Yuri B. Zudin

# Theory of Periodic Conjugate Heat Transfer

Second Edition



Springer

# Theory of Periodic Conjugate Heat Transfer

# Mathematical Engineering

## Series Editors:

Prof. Dr. Claus Hillermeier, Munich, Germany (volume editor)

Prof. Dr.-Ing. Johannes Huber, Erlangen, Germany

Prof. Dr. Albert Gilg, Munich, Germany

Prof. Dr. Stefan Schäffler, Munich, Germany

For further volumes:

<http://www.springer.com/series/8445>

Yuri B. Zudin

# Theory of Periodic Conjugate Heat Transfer

Second Edition

 Springer

Professor Dr.-Ing. habil. Yuri B. Zudin  
Russian State Atomic Energy Corporation  
Russian Nuclear-Power Machine Building  
Research Institute  
u. Kosmonavta Volkova 6a  
125171 Moskva  
Russia  
yzudin@gmail.com

ISBN 978-3-642-21420-2 e-ISBN 978-3-642-21421-9  
DOI 10.1007/978-3-642-21421-9  
Springer Heidelberg Dordrecht London New York

Library of Congress Control Number: 2011936129

© Springer-Verlag Berlin Heidelberg 2011

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer. Violations are liable to prosecution under the German Copyright Law.

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

*Cover design:* WMXDesign GmbH, Heidelberg

Printed on acid-free paper

Springer is part of Springer Science+Business Media ([www.springer.com](http://www.springer.com))

*This book is devoted to my children Maxim  
and Natalia, granddaughter Darya and  
grandson Alexey.*



# Preface

The material presented in this book crowns my long-term activity in the field of conjugate periodic heat transfer. Its first stage had passed under a scientific supervision of my teacher Professor Labuntsov (1929–1992), since publication in 1977 of our first article and finishing in 1984 with publishing our book in Russian: Labuntsov D.A., Zudin Y.B., *Processes of Heat Transfer with Periodic Intensity*. This stage was marked by defense in 1980 of my doctoral thesis: Zudin Y.B., *Analysis of Heat Transfer Processes with Periodic Intensity*. The subsequent period of interpreting the already gained results and accumulation of new knowledge had taken 7 years. Since 1991, I started a new cycle of publications on this subject, which was crowned in 2007 with the first edition of the present monograph. This stage was also marked with my habilitation (Zudin Y.B., *Approximate Theory of Heat Transfer Processes with Periodic Intensity*, 1996), as well as with fruitful scientific collaboration with my respected German colleagues Prof. U. Grigull, Prof. F. Mayinger, Prof. J. Straub, and Prof. T. Sattelmayer (TU München), Prof. W. Roetzel (Uni BW Hamburg), Prof. J. Mitrovic (Uni Paderborn), Prof. K. Stephan, Prof. M. Groll, and Prof. B. Weigand (Uni Stuttgart).

The objective of this monograph is to give an exhaustive answer for the question of how thermophysical and geometrical parameters of a body affect heat transfer characteristics under conditions of thermohydraulic pulsations. An applied objective of this book is to develop a universal method for the calculation of the average heat transfer coefficient for the periodic conjugate processes of heat transfer.

As a rule, it is possible to consider real “stationary” processes of heat transfer to be stationary only on the average. Actually, periodic, quasi-periodic, and various random fluctuations of parameters (velocities, pressure, temperatures, momentum and energy fluxes, vapor content, interphase boundaries, etc.) around their average values always exist in any type of fluid flow, except for purely laminar flows. Owing to the conjugate nature of the interface “fluid flow–streamlined body,” both fluctuation and average values of temperatures and heat fluxes on the heat transfer surface generally depend on thermophysical and geometrical characteristics of the heat transferring wall.



In this connection, a principle question arises about the possible influence of the material and the thickness of the wall on the key parameter of convective heat transfer, namely, heat transfer coefficient. The facts of such an influence were earlier noticed in experimental investigations of heat transfer at nucleate boiling, dropwise condensation, as well as in some other cases. In these studies, heat transfer coefficients determined as a ratio of the average heat flux on the surface and the average temperature difference “wall-fluid” could differ noticeably for various materials of the wall (and also for its different thicknesses).

In 1977, a concept of a true heat transfer coefficient was proposed for the first time in the work of Labuntsov and Zudin. According to this concept, actual values of the heat transfer coefficient (for each point of the heat transferring surface and at each moment of time) are determined solely by hydrodynamic characteristics of the fluid flow and consequently do not depend on parameters of the body. Fluctuations of parameters occurring in the fluid flow will result in respective fluctuations of the true heat transfer coefficient also independent of the material and thickness of the wall. Then, from a solution of the heat conduction equation with a boundary condition of the third kind, it is possible to find a temperature field in the body (and, hence, on the heat transfer surface) and, as a result, to calculate the required experimental heat transfer coefficient as a ratio of an average heat flux to an average temperature difference. This value (determined in traditional heat transfer experiments and used in applied calculations) should in general case depend on the conjugation parameters.

A study of interrelations of the heat transfer coefficients averaged based on different procedures (true and experimental) laid the foundation of the first edition of this book. A fundamental result obtained in this book was that the average experimental value of the heat transfer coefficient is always less than the average true value of this parameter.

The first edition included the following seven chapters.

Chapter 1 presented a qualitative description of the method for investigations of periodic conjugate convective–conductive problems “fluid flow–streamlined body.” An analysis of physical processes representing heat transfer phenomena with periodic fluctuations was also performed.

In Chap. 2, a boundary problem for the twodimensional unsteady heat conduction equation with a periodic boundary condition of the third kind was analyzed. To characterize the thermal effects of a solid body on the average heat transfer, a concept of a *factor of conjugation* was introduced. It was shown that the quantitative effect of the conjugation in the problem can be rather significant.

Chapter 3 represented a general solution design for a boundary problem for the equation of heat conduction with a periodic boundary condition of the third kind. Analytical solutions were obtained for the characteristic laws of variation of the true heat transfer coefficient, namely, harmonic, inverse harmonic, stepwise, and delta-like.

In Chap. 4, a universal algorithm of a general approximate solution of the problem was developed. On its basis, solutions were obtained for a series of problems at different laws of periodic fluctuations of the true heat transfer coefficient.

Chapter 5 dealt with conjugate periodic heat transfer for “complex” cases of external heat supply: heat transfer at a contact either with environment or with a second body. A generalized solution for the factor of conjugation for the bodies of the “standard form” was obtained. A problem of conjugate heat transfer for a case of bilateral periodic heat transfer was also investigated in this chapter.

In Chap. 6, an analysis was given for the cases of asymmetric and nonperiodic fluctuations of the true heat transfer coefficient.

Chapter 7 included some applied problems of the periodic conjugate heat transfer theory such as jet impingement onto a surface, dropwise condensation, and nucleate boiling.

In Appendix A, proofs were presented of some properties of the two-dimensional unsteady equation of heat conduction with a periodic boundary condition of the third kind. Consequences of these proofs allowed establishing limiting values of the factor of conjugation.

Appendix B represented a study of the eigenfunctions of the solution for the two-dimensional unsteady equation of heat conduction obtained by the method of separation of variables.

In Appendix C, the problem of convergence of infinite chain fractions was considered. A generalization of the proof of the third theorem of Khinchin for the case where the terms in the fraction possess a negative sign was obtained using the method of mathematical induction.

In Appendix D, a proof of divergence of infinite series obtained in Chap. 3 for the particular solution of the heat conduction equation was documented.

Appendix E dealt with an investigation of eigenfunctions of the heat conduction equation solution for complex cases of the external heat supply considered in Chap. 5.

The second edition, which includes (without any changes) the material of the first edition, was completed with two additional chapters and two appendix.

Chapter 8 is devoted to an investigation into effects of the thermophysical parameters and the channel wall thickness on hydrodynamic instability of the type called “density waves.” The boundary of stability of fluid flow in a channel at supercritical pressures was found analytically. As an application, the problem was considered dealing with maintenance of effective functioning of the thermostating system for superconducting magnets.

In Chap. 9, an analytical method is outlined for heat transfer calculation in turbulent channel flow at supercritical pressures. This method allows considering effects of varying thermophysical properties of fluid on heat transfer coefficient averaged over the period of turbulent pulsations.

Appendix F is devoted to phase transitions in the area of nanoscopic scales. A periodic quantum mechanical model is offered for the process of homogeneous nucleation.

Appendix G deals with determining one of the important parameters of periodic two-phase flows, which is the rise velocity of the Taylor bubbles in round pipes.

I am deeply grateful to Prof. Wilfried Roetzel (Uni Bundeswehr Hamburg), the meeting with whom in 1995 served as a starting point in planning this

book and formation of its ideology. I used each subsequent stay in Germany for fruitful discussions with Prof. Roetzl, which have substantially helped me in the preparation of the book. In 2005, my collaborative work with Prof. Bernhard Weigand (Uni Stuttgart) has begun who has actively supported my idea to write a book and repeatedly invited me to visit the Institute of Aerospace Thermodynamics to perform joint research. During our numerous discussions, Prof. Weigand has made a number of useful comments and suggestions, which have considerably improved the content of the book.

I am very much grateful to Dr. Habil. Claus E. Ascheron (Senior Editor, Physics, Springer – Materials Science; Condensed Matter and Solid State Physics; Biological and Medical Physics; Biophysics) for his keen interest to the publication of this book and his effort toward its successful advancement on the book market.

I am also warmly thankful to Dr. Igor V. Shevchuk (MBtech Group GmbH & Co. KGaA) for his very useful comments, which contributed much toward considerable improvement of the scientific translation of the book manuscript into English.

Reading of the books of B. Weigand, “Analytical Methods for Heat Transfer and Fluid Flow Problems,” 2004, and I.V. Shevchuk, “Convective Heat and Mass Transfer in Rotating Disk Systems,” 2009, recently published by the Springer Verlag helped me significantly in the selection of the new material for the second edition of my book.

The publication of both the first and the second editions of the book would have been impossible without the long-term financial support of my activity in German universities (TU München, Uni Paderborn, Uni Stuttgart, Uni Bundeswehr Hamburg) from the German Academic Exchange Service (DAAD), which I very gratefully acknowledge. Being happy fivefold (!) grantee of the DAAD, I would like to express my sincere gratitude to the people who have made it possible: Dr. W. Trenn, Dr. P. Hiller, Dr. T. Prah, Dr. G. Berghorn, Dr. H. Finken, and also to all other DAAD employees both in Bonn and in Moscow.

I express my special gratitude to my wife Tatiana who always served me as an invaluable moral support in my life-long scientific activity. For my academic degree of Prof. Dr.-Ing. Habil. and also for the appearance of the first and the second editions of my book, I am greatly obliged to my beloved spouse.

I dare to hope that the second edition of my book will be so favorably accepted by readers, as the first one.

Stuttgart  
August, 2011

*Y.B. Zudin*

# Contents

<b>1</b>	<b>Introduction</b> .....	1
1.1	Heat Transfer Processes Containing Periodic Oscillations .....	1
1.1.1	Oscillation Internal Structure of Convective Heat Transfer Processes .....	1
1.1.2	Problem of Correct Averaging the Heat Transfer Coefficients .....	3
1.2	Physical Examples .....	6
1.3	Numerical Modeling of Conjugate Convective–Conductive Heat Transfer .....	10
1.4	Mechanism of Hydrodynamic Oscillations in a Medium Flowing Over a Body .....	11
1.4.1	Van Driest Model .....	11
1.4.2	Periodic Model of the Reynolds Analogy .....	13
1.4.3	Model of Periodical Contacts .....	15
1.5	Hydrodynamic HTC .....	17
1.6	Previous Investigations of Heat Transfer Processes with Periodic Intensity .....	19
1.7	Analytical Methods .....	19
	References .....	20
<b>2</b>	<b>Construction of a General Solution of the Problem</b> .....	25
2.1	Boundary Value Problem for the Heat Conduction Equation.....	25
2.2	Spatial and Temporal Types of Oscillations .....	28
2.3	Interrelation Between the Two Averaged Coefficients of Heat Transfer.....	29
2.4	Dimensionless Parameters .....	32
2.5	Factor of Conjugation (FC): An Analysis of Limiting Variants .....	33
	References .....	34
<b>3</b>	<b>Solution of Characteristic Problems</b> .....	37
3.1	Construction of the General Solution .....	37
3.2	Harmonic Law of Oscillations .....	39

3.3	Inverse Harmonic Law of Oscillations .....	45
3.4	Delta-Like Law of Oscillations .....	53
3.5	Step Law of Oscillations .....	55
3.6	Comparative Analysis of the Conjugation Effects (Smooth and Step Oscillations) .....	69
3.7	Particular Exact Solution .....	70
	References .....	71
<b>4</b>	<b>Universal Algorithm of Computation of the Factor of Conjugation...</b>	<b>73</b>
4.1	Smooth Oscillations (Approximate Solutions) .....	73
4.2	BC on a Heat Transfer Surface (Series Expansion in a Small Parameter) .....	75
4.3	Derivation of a Computational Algorithm .....	77
4.4	Phase Shift Between Oscillations .....	80
4.5	Method of a Small Parameter .....	83
4.6	Application of the Algorithm for an Arbitrary Law of Oscillations .....	85
4.7	Filtration Property of the Computational Algorithm .....	91
4.8	Generalized Parameter of the Thermal Effect .....	92
4.9	Advantages of the Computational Algorithm .....	93
	References .....	93
<b>5</b>	<b>Solution of Special Problems</b> .....	<b>95</b>
5.1	Complex Case of Heating or Cooling .....	95
5.2	Heat Transfer on the Surface of a Cylinder .....	102
5.3	Heat Transfer on the Surface of a Sphere .....	103
5.4	Parameter of Thermal Effect for Different Geometrical Bodies .....	104
5.5	Overall ATHTC .....	106
	5.5.1 Overall EHTC .....	106
	5.5.2 Bilateral Spatiotemporal Periodicity of Heat Transfer (A Qualitative Analysis) .....	109
	References .....	111
<b>6</b>	<b>Step and Nonperiodic Oscillations of the Heat Transfer Intensity</b> ....	<b>113</b>
6.1	Asymmetric Step Oscillations .....	113
6.2	Nonperiodic Oscillations .....	119
	References .....	122
<b>7</b>	<b>Practical Applications of the Theory</b> .....	<b>123</b>
7.1	Model Experiment .....	123
7.2	Dropwise Condensation .....	124
7.3	Nucleate Boiling .....	129
	7.3.1 Theory of Labuntsov .....	129
	7.3.2 Periodic Model of Nucleate Boiling .....	131
	References .....	138

<b>8</b>	<b>Wall's Thermal Effect on Hydrodynamic Flow Stability</b> .....	141
8.1	Flow of a Liquid with Supercritical Parameters in a Heated Channel .....	141
8.2	Density Wave Instability Phenomena .....	142
8.3	Scenario of Evolution of the Density Wave Instability .....	148
8.4	Basic Equations of the Density Wave Instability .....	149
8.5	Computation of Wall's Thermal Effect .....	150
8.6	Analytical Problem Solution .....	151
	References .....	158
<b>9</b>	<b>Periodical Model of Turbulent Heat Transfer</b> .....	159
9.1	Surface Rejuvenation Model .....	159
9.1.1	Investigation of Turbulent Heat Transfer by Means of the Bursting Effect .....	159
9.1.2	Variable Thermophysical Properties of Fluid .....	160
9.2	Method of Relative Correspondence .....	162
9.3	Integral Methods for Simulation of Heat and Mass Transfer in a Boundary Layer .....	165
9.4	Differential Equation for the Heat Flux .....	167
9.4.1	Exact Solution .....	167
9.4.2	Approximate Analytical Solution .....	168
9.4.3	Validation of the Method. Laminar Boundary Layer .....	171
9.5	Supercritical Pressures Region .....	173
9.5.1	Thermophysical Properties .....	173
9.5.2	Generalization of the Results .....	176
	References .....	178
<b>A</b>	<b>Proof of the Fundamental Inequalities</b> .....	181
A.1	Proof of the First Fundamental Inequality .....	181
A.2	Proof of the Second Fundamental Inequality .....	186
<b>B</b>	<b>Functions of Thickness</b> .....	189
B.1	Spatial Type of Oscillations .....	190
B.2	Temporal Type of Oscillations .....	190
<b>C</b>	<b>Infinite Chain Fractions</b> .....	193
C.1	Fundamental Theorems of Khinchin .....	193
C.2	Generalization of the Third Theorem of Khinchin .....	194
<b>D</b>	<b>Proof of Divergence of the Infinite Series</b> .....	197
D.1	Spatial Type of Oscillations .....	197
D.2	Temporal Type of Oscillations .....	198
<b>E</b>	<b>Functions of Thickness for Special Problems</b> .....	201
E.1	Heat Transfer from the Ambience .....	201
E.2	Heat Transfer from an External Semi-infinite Body .....	202

- F Periodicity in Nanoscopic Scale** ..... 203
- References ..... 207
  
- G Rise Velocity of a Taylor Bubble in a Round Tube** ..... 209
- G.1 Solutions of Prandtl and Taylor ..... 209
- G.2 Correct Approximate Statement of the Problem ..... 211
- G.3 Flow in the Vicinity of the Critical Point ..... 215
- G.4 Solution by the Method of Collocations ..... 217
- G.5 Asymptotical Solution ..... 218
- G.6 Plane Taylor Bubble ..... 221
- References ..... 223
  
- Index** ..... 225

# List of Abbreviations

ATHTC	Averaged True Heat Transfer Coefficient
BC	Boundary Condition
EHTC	Experimental Heat Transfer Coefficient
FC	Factor of Conjugation
HN	Homogeneous nucleation
HTC	Heat Transfer Coefficient
MRC	Method of Relative Correspondence
PTE	Parameter of the Thermal Effect
SCP	Supercritical pressures
SRM	Surface rejuvenation model
TBC	Thermal Boundary Conditions
THTC	True Heat Transfer Coefficient





# List of Symbols

$A_k, A_k^*$ (–)	Complex conjugate eigenvalues
$B_k, B_k^*$ (–)	Complex conjugate eigenfunctions
$b$ (–)	Amplitude of oscillations of the true heat transfer coefficient
$C_f/2$ (–)	Friction factor
$c$ [J/(kg K)]	Specific heat
$d_0$ (m)	Nozzle diameter
$F_k$ (–)	Real parts of eigenfunctions
$h$ [W/(m <sup>2</sup> K)]	True heat transfer coefficient (THTC)
$\langle h \rangle$ [W/(m <sup>2</sup> K)]	Averaged true heat transfer coefficient (ATHTC)
$\bar{h}$ (–)	Dimensionless averaged true heat transfer coefficient or Biot number
$h_m$ [W/(m <sup>2</sup> K)]	Experimental heat transfer coefficient (EHTC)
$h_0$ [W/(m <sup>2</sup> K)]	Steady state heat transfer coefficient
$\bar{h}_0$ (–)	Dimensionless stationary heat transfer coefficient
$h_{fg}$ (J/kg)	Specific enthalpy of evaporation
$I_n$ (–)	Imaginary parts of eigenfunctions
$Ja$ (–)	Jacob number
$k$ [W/(m K)]	Thermal conductivity
$L$ (m)	Distance between nucleate boiling sites
$m$ (–)	Inverted Fourier number
$n_F$ (m <sup>-2</sup> )	Number of boiling sites
$K$ (–)	Ratio of thermal potentials of contacting media
$p$ (Pa)	Pressure
$Pr$ (–)	Prandtl number
$q$ (W/m <sup>2</sup> )	Heat flux density
$\langle q \rangle$ (W/m <sup>2</sup> )	Averaged heat flux density
$\hat{q}$ (W/m <sup>2</sup> )	Oscillating heat flux density

$q_V$ ( $\text{W}/\text{m}^3$ )	Volumetric heat source
$R_n$ (—)	Real parts of eigenvalues
$R_*$ (m)	Critical radius of vapor nucleus
$St$ (—)	Stanton number
$t$ (—)	Dimensionless time
$T_s$ (K)	Saturation temperature
$u$ (m/s)	Velocity
$u_0$ (m/s)	Free stream velocity
$u_*$ (m/s)	Friction velocity
$U$ [ $\text{W}/(\text{m}^2 \text{K})$ ]	Overall heat transfer coefficient
$\langle U \rangle$ [ $\text{W}/(\text{m}^2 \text{K})$ ]	Averaged true overall heat transfer coefficient
$U_m$ [ $\text{W}/(\text{m}^2 \text{K})$ ]	Experimental overall heat transfer coefficient
$\langle \bar{U} \rangle$ (—)	Dimensionless averaged true overall heat transfer coefficient
$E$ (—)	Generalized factor of conjugation
$X$ (m)	Spanwise coordinate
$x$ (—)	Dimensionless spanwise coordinate
$Z$ (m)	Coordinate along the surface of heat transfer
$Z_0$ (m)	Spatial periods of oscillation
$z$ (—)	Dimensionless coordinate along the heat transfer surface

### Greek Letter Symbols

$\alpha$ ( $\text{m}^2/\text{s}$ )	Thermal diffusivity
$\Gamma$ ( $\text{N}/\text{m}^2$ )	Shear stress
$\delta$ (m)	Wall thickness (flat plate)
$\bar{\delta}$ (—)	Dimensionless wall thickness (flat plate)
$\delta_f$ (m)	Thickness of liquid film
$\varepsilon$ (—)	Factor of conjugation (FC)
$\vartheta$ (K)	Temperature
$\langle \vartheta \rangle$ (K)	Averaged temperature
$\hat{\vartheta}$ (K)	Oscillating temperature
$\vartheta_0$ (K)	Free stream temperature
$\vartheta^\bullet$ (K/m)	Gradient of oscillating temperature or dimensionless heat flux density
$\vartheta_\Sigma$ (K)	Total temperature difference in the three-part system
$\theta$ (—)	Dimensionless oscillation temperature
$\theta^\bullet$ (—)	Dimensionless gradient of the oscillation temperature (or dimensionless heat flux density)
$\xi$ (—)	Generalized coordinate of a progressive wave

$\xi_{\vartheta}$ (–)	Phase shift between oscillation of true heat transfer coefficient and temperature
$\xi_q$ (–)	Phase shift between oscillation of true heat transfer coefficient and heat flux
$\mu$ [kg/(m s)]	Dynamic viscosity
$\nu$ (m <sup>2</sup> /s)	Kinematic viscosity
$\rho$ (kg/m <sup>3</sup> )	Density
$\sigma$ (N/m)	Surface tension
$\tau$ (s)	Time
$\tau_0$ (s)	Time period of oscillation
$\Phi_k$ (–)	Imaginary parts of eigenfunctions
$\chi$ (–)	Parameter of thermal effect (PTE)
$\psi$ (–)	Periodic part of the heat transfer coefficient
$\omega$ (s <sup>-1</sup> )	Frequency

### Subscripts

+	Active period of heat transfer
–	Passive period of heat transfer
f	Fluid
g	Gas
0	External surface of a body (at $X = 0$ )
$\delta$	Heat transfer surface (at $X = \delta$ )
min	Minimal value
max	Maximal value
w	Another (second) body

### Definition of Nondimensional Numbers and Groups

$\langle \bar{h} \rangle = \langle h \rangle Z_0 / k$	Dimensionless averaged true heat transfer coefficient or Biot number
$\bar{h}_0 = h_0 Z_0 / k$	Dimensionless stationary heat transfer coefficient
$E = U_m / \langle U \rangle$	Generalized factor of conjugation
$Ja = \rho_f c_{pf} \vartheta / \rho_g h_{fg}$	Jacob number
$K = \sqrt{k} c \rho / k_f c_f \rho_f$	Ratio of thermal potentials of the contacting media
$m = Z_0^2 / \alpha \tau_0$	Inverted Fourier number
$Pr = \nu_f / \alpha_f$	Prandtl number
$St = q / \rho_f c_f u_0 \vartheta_0$	Stanton number
$\bar{U}_m = U_m Z_0 / k$	Dimensionless averaged true overall heat transfer coefficient

$\bar{U}_m = U_m Z_0 / k$	Dimensionless experimental overall heat transfer coefficient
$\bar{\delta} = \delta / Z_0$	Dimensionless wall thickness (flat plate)
$\varepsilon = h_m / \langle h \rangle$	Factor of conjugation

# Chapter 1

## Introduction

### 1.1 Heat Transfer Processes Containing Periodic Oscillations

#### *1.1.1 Oscillation Internal Structure of Convective Heat Transfer Processes*

Real stationary processes of heat transfer, as a rule, can be considered stationary only on the average. Actually (except for the purely laminar cases), flows are always subjected to various periodic, quasi-periodic, and other casual oscillations of velocities, pressure, temperatures, momentum and energy fluxes, vapor content and interphase boundaries about their average values. Such oscillations can be smooth and periodic (wave flow of a liquid film or vapor, a flow of a fluctuating coolant over a body), sharp and periodic (hydrodynamics and heat transfer at slug flow of a two-phase media in a vertical pipe; nucleate and film boiling process), one can have complex stochastic character (turbulent flows). Oscillations of parameters have in some cases spatial nature, and in others they are temporal, and generally one can say that the oscillations have mixed spatiotemporal character.

The theoretical base for studying instantly oscillations and at the same time stationary on the average heat transfer processes are the unsteady differential equations of momentum and energy transfer, which in the case of two-phase systems can be notated for each of the phases separately and be supplemented by transmission conditions (transmission conditions). An exhaustive solution of the problem could be a comprehensive analysis with the purpose of a full description of any particular fluid flow and heat transfer pattern with all its detailed characteristics, including various fields of oscillations of its parameters.

However, at the time being such an approach cannot be realized in practice. The problem of modeling turbulent flows [1] can serve as a vivid example. As a rule at its theoretical analysis, Reynolds-averaged Navier–Stokes equations are considered, which describe time-averaged quantities of fluctuating parameters, or in other words turbulent fluxes of the momentum and energy. To provide a closed

description of the process, these correlations by means of various semiempirical hypotheses are interrelated with time-averaged fields of velocities and enthalpies. Such schematization results in the statement of a stationary problem with spatially variable coefficients of viscosity and thermal conductivity. Therefore, as boundary conditions here, it is possible to set only respective stationary conditions on the heat transfer surface of such a type as, for example, “constant temperature,” “constant heat flux.”

It is necessary to specially note that the replacement of the full “instant” model description with the time-averaged one inevitably results in a loss of information on the oscillations of fluid flow and heat transfer parameters (velocities, temperatures, heat fluxes, pressure, friction) on a boundary surface. Thus, the theoretical basis for an analysis of the interrelation between the temperature oscillations in the flowing ambient medium and in the body is omitted from the consideration. And generally, the problem of an account for possible influence of thermophysical and geometrical parameters of a body on the heat transfer at such an approach becomes physically senseless. For this reason, such a “laminarized” form of the turbulent flow description is basically not capable of predicting and explaining the wall effects on the heat transfer characteristics, even if these effects are observed in practice. The problem becomes especially complicated at imposing external oscillations on the periodic turbulent structure that takes place, in particular, flows over aircraft and spacecraft. Unresolved problems of closing the Navier–Stokes equations in combination with difficulties of numerical modeling make a problem of detailed prediction of a temperature field in the flowing fluid very complicated. In some cases, differences between the predicted and measured local “*heat transfer coefficient*” (HTC) exceeds 100%.

In this connection, the direction in the simulation of turbulent flows based on the use of the primary transient equations [2] represents significant interest. This book represents results of numerical modeling of the turbulent flows in channels subjected to external fields of oscillations (due to vortical generators, etc.). It is shown that in this case an essentially anisotropic and three-dimensional flow pattern emerges strongly different from that described by the early theories of turbulence [1]. In the near-wall zone, secondary flows in the form of rotating “vortical streaks” are induced that interact with the main flow. As a result, oscillations of the thermal boundary layer thickness set on, leading to periodic enhancement or deterioration of heat transfer. Strong anisotropy of the fluid flow pattern results in the necessity of a radical revision of the existing theoretical methods of modeling the turbulent flows. Hence, for example, the turbulent Prandtl number being in early theories of turbulence [1] a constant of the order of unity (or, at the best, an indefinite scalar quantity) becomes a tensor.

It is necessary to emphasize that all the mentioned difficulties are related to the nonconjugated problem when the role of a wall is reduced only to maintenance of a “boundary condition” (BC) on the surface between the flowing fluid and the solid wall.

### 1.1.2 Problem of Correct Averaging the Heat Transfer Coefficients

The basic applied task of this book is the investigation into the effects of a body (its thermophysical properties, linear dimensions, and geometrical configuration) on the traditional HTC, measured in experiments and used in engineering calculations. Processes of heat transfer are considered stationary on average and fluctuating instantly. A new method for investigating the conjugate problem “fluid flow - body” is presented. The method is based on a replacement of the complex mechanism of oscillations of parameters in the flowing coolant by a simplified model employing a varying “true heat transfer coefficient” specified on a heat transfer surface.

The essence of the developed method can be explained rather simply. Let us assume that we have perfect devices measuring the instant local values of temperature and heat fluxes at any point of the fluid and heated solid body. Then the hypothetical experiment will allow finding the fields of temperatures and heat fluxes and their oscillations in space and in time, as well as their average values and all other characteristics. In particular, it is possible to present the values of temperatures (exactly saying, temperature heads or loads, i.e., the temperatures counted from a preset reference level) and heat fluxes on a heat transfer surface in the following form:

$$\vartheta = \langle \vartheta \rangle + \hat{\vartheta}, \quad (1.1)$$

$$q = \langle q \rangle + \hat{q}, \quad (1.2)$$

i.e., to write them as the sum of the averaged values and their temporal oscillations. For the general case of spatiotemporal oscillations of characteristics of the process, the operation of averaging is understood here as a determination of an average with respect to time  $\tau$  and along the heat transferring surface (with respect to the coordinate  $Z$ ). The “true heat transfer coefficient” (THTC) is determined on the basis of (1.1–1.2) according to Newton’s law of heat transfer [3, 4]:

$$h = \frac{q}{\vartheta}. \quad (1.3)$$

This parameter can always be presented as a sum of an averaged part and a fluctuating additive:

$$h = \langle h \rangle + \hat{h}. \quad (1.4)$$

It follows from here that the correct averaging of the HTC is as follows:

$$\langle h \rangle = \left\langle \frac{q}{\vartheta} \right\rangle. \quad (1.5)$$

Therefore we shall call parameter  $\langle h \rangle$  an “averaged true heat transfer coefficient” (ATHTC). The problem consists in the fact that the parameter  $\langle h \rangle$  cannot be directly used for applied calculations, since it contains initially the unknown information



of oscillations  $\hat{\vartheta}, \hat{q}$ . This fact becomes evident if (1.5) is rewritten with the help of (1.1–1.2):

$$\langle h \rangle = \left\langle \frac{\langle q \rangle + \hat{q}}{\langle \vartheta \rangle + \hat{\vartheta}} \right\rangle. \quad (1.6)$$

The purpose of the heat transfer experiment is the measurement of averaged values on averaged temperature  $\langle \vartheta \rangle$  and a heat flux  $\langle q \rangle$  on the surfaces of a body and determination of the traditional HTC

$$h_m = \frac{\langle q \rangle}{\langle \vartheta \rangle}. \quad (1.7)$$

The parameter  $h_m$  is fundamental for carrying out engineering calculations, designing heat transfer equipment, composing thermal balances, etc. However, it is necessary to point out that transition from the initial Newton's law of heat transfer (1.3) to the restricted (1.7) results in the loss of the information of the oscillations of the temperature  $\hat{\vartheta}$  and the heat fluxes  $\hat{q}$  on the wall.

Thus, it is logical to assume that the influence of the material and the wall thickness of the body taking part in the heat transfer process on HTC  $h_m$  uncovered in experiments is caused by noninvariance of the value of  $h_m$  with respect to the Newton's law of heat transfer. For this reason, we shall refer further to the parameter  $h_m$  as to an "experimental heat transfer coefficient" (EHTC).

Thus, we have two alternative procedures of averaging the HTC: true (1.5) and experimental (1.7). The physical reason of the distinction between  $\langle h \rangle$  and the  $h_m$  can be clarified with the help of the following considerations:

- Local values  $\langle \vartheta \rangle$  and  $\langle q \rangle$  on a surface where heat transfer takes place are formed as a result of the thermal contact of the flowing fluid and the body.
- Under conditions of oscillations of the characteristics of the coolant, temperature oscillations will penetrate inside the body.
- Owing to the conjugate nature of the heat transfer in the considered system, both fluctuating  $\hat{\vartheta}, \hat{q}$  and averaged  $\langle \vartheta \rangle, \langle q \rangle$  parameters on the heat transfer surface depend on the thermophysical and geometrical characteristics of the body.
- The ATHC  $\langle h \rangle$  directly follows from Newton's law of heat transfer (1.3) (which is valid also for the unsteady processes) and consequently it is determined by hydrodynamic conditions in the fluid flowing over the body.
- The EHTC  $h_m$  by definition does not contain the information on oscillations  $\hat{\vartheta}, \hat{q}$ , and consequently it is in the general case a function of parameters of the interface between fluid and solid wall.
- Aprioristic denying of dependence of the EHTC on material properties and wall thickness is wrong, though under certain conditions quantitative effects of this influence might be insignificant.

From the formal point of view, the aforementioned differences between the true (1.5) and experimental (1.7) laws of averaging of the actual HTC is reduced to