Koenraad Van Schuylenbergh
Robert Puers

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Inductive Powering

Basic Theory and Application to Biomedical Systems

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Inductive Powering

Basic Theory and Application to Biomedical Systems

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Preface

This book is based on research carried out at the Katholieke Universiteit Leuven, Belgium, on transferring data and power over poorly coupled inductive links with near theoretical energy efficiency. This largely mathematical study grew out of the frustration that the link circuits designed with methods available in literature did not perform as calculated. This is attributed to the poor coil coupling typically encountered in our applications. This triggered our eagerness to dive into the theory and find out why things went astray.

Inductive powering has been a reliable and simple method for years to wirelessly transfer power and data over short distances. This engineering discipline originated in biotelemetry with cochlear implants as the commercially most common application, and is now also widely applied in radio-frequency identification (RFID), with high-volume applications like wireless ID tags for asset tracking, key cards for building access control, electronic passports, implanted ID tags for husbandry control etc…

The field of inductive powering splits up along two orthogonal axis: the amount of transferred power and the coil coupling which directly relates to transfer range. Biotelemetry applications typically transfer a few milliwatts over a few centimetres. The RFID applications are characterised by transfer ranges up to half a metre, but run at microwatt power levels where energetic link efficiency is of little importance. The present study aims at transferring several milliwatts over longer ranges with near the theoretical efficiency limit by a careful optimisation of the driven inductive link (chapter 5) and by servo control of the driver output to the actual needs (chapter 6). Although this textbook emphasises applications with weak coil coupling, the described methods and formulae are universal to the complete range of coil coupling factors.

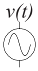
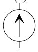
Six chapters make up this book:

- **Chapter 1** is a general introduction on telemetry to situate inductive coupling amongst its competitors.
- **Chapter 2** outlines the basics of inductive powering. It starts off with a summary of magnetic induction theory and then delves into the existing methods to design inductive links. This allows pin pointing the problems that crop up with poor coil coupling. It shows that the approximate formulae that are commonly used to optimise coil sets become invalid at low coupling and also that the design of the coil driver amplifier must be included in the optimisation process.
  This observation dictates the structure of the following chapters.
- A set of exact design formulae that describe the coil link is first derived in **chapter 3**.
- **Chapter 4** then studies the design of primary coil drivers.
- **Chapter 5** finally develops a step-by-step design procedure for driven inductive links that operate as close as possible to their theoretical efficiency limit.
- Basic inductive links may be finicky to handle in many real-world situations. **Chapter 6** therefore expands the optimised driven link with servo loops that automatically adjust the link circuit to changing coupling and load conditions.
- This book also provides three **appendices**. There is a short overview of vector mathematics. A second appendix rounds up some tips and tricks on coil models and measurements. The third appendix is a mathematical exposé on saturating-class-C amplifiers.

Koenraad Van Schuylenbergh
Robert Puers
January 2009.

Symbols and units

Symbol	Description	Unit	
$=$	is equal to		
$\equiv$	is defined as		
$\approx$	is approximately equal to		
$\propto$	is linearly proportional to		
Complex numbers			
$Re\{\underline{X}\}$	The real part of the complex number $\underline{X}$		
$Im\{\underline{X}\}$	The imaginary part of the complex number $\underline{X}$		
$\lvert\underline{X}\rvert$	The magnitude of the complex number $\underline{X}$		
$\angle\underline{X}$	The phase angle of the complex number $\underline{X}$		
Electromagnetic fields			
A	surface	m	
$\vec{E}$	electric field strength	V/m	
$\vec{D}$	electric displacement: $\vec{D} = \varepsilon\vec{E} = \varepsilon_0\vec{E} + \vec{P}$	C/m^2	
$\vec{P}$	medium polarisation	C/m^2	
$\vec{J}$	electric current density	A/m^2	
ρ	electric charge density	C/m^3	
Φ_B	magnetic flux	Wb	$=T.m^2$
$\vec{B}$	magnetic flux density: $\vec{B} = \mu\vec{H} = \mu_0\left(\vec{H} + \vec{M}\right)$	T	$= \frac{N}{A.m}$
$\vec{H}$	magnetic field strength	A/m	
$\vec{M}$	medium magnetisation	A/m	
σ	medium conductivity	S/m	$= \frac{1}{\Omega.m}$
ε	medium dielectric constant	F/m	
μ	medium permeability	H/m	
μ_r	relative medium permeability $\equiv \mu/\mu_0$	-	
R_m	reluctance or magnetic resistance	A/Wb	

Symbol	Description	Unit
Waves		
t	time	s
λ	wavelength	m
T	period	s
f	frequency= $1/T$	Hz $= s^{-1}$
ω	angular or radian frequency	rad/s
Voltages		
$v(t)$	a voltage signal	V
	an ideal sinusoidal voltage source	
$\hat{V}$	peak amplitude of a sinusoidal voltage signal: $v(t)=\hat{V}.\cos(\omega t+\varphi)=\mathrm{Re}\left\{\hat{V}.e^{j(\omega t+\varphi)}\right\}=\mathrm{Re}\left\{\underline{\hat{V}}.e^{j\omega t}\right\}$	V
$\underline{\hat{V}}$	complex peak amplitude of a sinusoidal voltage signal: $\underline{\hat{V}}=\hat{V}.e^{j\varphi}$	V
V	RMS amplitude of a sinusoidal voltage signal: $V=\hat{V}/\sqrt{2}$	V
$\underline{V}$	complex RMS amplitude of a sinusoidal voltage signal: $\underline{V}=\underline{\hat{V}}/\sqrt{2}=V.e^{j\varphi}$	V
φ	phase of a sinusoidal voltage signal	rad
Currents		
$i(t)$	a current signal	A
	an ideal sinusoidal current source	
$\hat{I}$	peak amplitude of a sinusoidal current signal: $i(t)=\hat{I}.\cos(\omega t+\psi)=\mathrm{Re}\left\{\hat{I}.e^{j(\omega t+\psi)}\right\}=\mathrm{Re}\left\{\underline{\hat{I}}.e^{j\omega t}\right\}$	A
$\underline{\hat{I}}$	complex peak amplitude of a sinusoidal current signal: $\underline{\hat{I}}=\hat{I}.e^{j\psi}$	A
I	RMS amplitude of a sinusoidal current signal: $I=\hat{I}/\sqrt{2}$	A
$\underline{I}$	complex RMS amplitude of a sinusoidal current signal: $\underline{I}=\underline{\hat{I}}/\sqrt{2}=I.e^{j\psi}$	A
ψ	phase of a sinusoidal current signal	rad
The primary coil		
$i_1(t)$	primary coil current (through the ideal coil of the series R-L-C model)	A
$v_1(t)$	primary coil voltage (across the ideal coil of the series R-L-C model)	V
n_1	number of primary coil windings	-
r_1	primary coil radius	m
L_1	primary coil inductance (R-L coil model)	H $=\frac{V.s}{A}$
R_{L_1}	parasitic primary coil resistance (R-L coil model)	Ω $= V/A$
Q_{L_1}	unloaded primary coil quality factor (R-L coil model) $Q_{L_1}=\frac{\omega L_1}{R_{L_1}}$	-
L_{s_1}	primary coil inductance (series R-L-C coil model)	H
R_{s_1}	Parasitic primary coil resistance (series R-L-C coil model)	Ω
C_{s_1}	Parasitic primary coil capacitor (series R-L-C coil model)	F
$Q_{L_{S_1}}$	unloaded primary coil quality factor (series R-L-C coil model) $Q_{L_{S_1}}=\frac{\omega L_{S_1}}{R_{S_1}}$	-

Symbol	Description	Unit
The secondary coil		
$i_2(t)$	secondary coil current (through the ideal coil of the series R-L-C model)	A
$v_2(t)$	secondary coil voltage (across the ideal coil of the series R-L-C model)	V
n_2	number of secondary coil windings	-
r_2	secondary coil radius	m
L_2	secondary coil inductance (R-L coil model)	H
R_{L_2}	parasitic secondary coil resistance (R-L coil model)	Ω
Q_{L_2}	secondary coil quality factor (R-L coil model) $Q_{L_2} = \frac{\omega L_2}{R_{L_2}}$	-
L_{s_2}	secondary coil inductance (series R-L-C coil model)	H
R_{s_2}	parasitic secondary coil resistance (series R-L-C coil model)	Ω
C_{s_2}	parasitic secondary coil capacitor (series R-L-C coil model)	F
$Q_{L_{S_2}}$	secondary coil quality factor (series R-L-C coil model) $Q_{L_{S_2}} = \frac{\omega L_{S_2}}{R_{L_{S_2}}}$	-
The coupling		
d	distance between the coil planes for parallel coils	m
M	mutual inductance	H
k	coil coupling factor (also called coupling coefficient) $k = \frac{M}{\sqrt{L_{s_1}.L_{s_2}}}$	-
n	square root ratio of the coil inductances $n = \sqrt{\frac{L_{s_2}}{L_{s_1}}}$	-
Loads		
R_{out}	d.c. load at the regulated output of the inductive link	Ω
$R_{d.c.}$	equivalent d.c. resistance of the loaded regulator	Ω
R_{load_2}	equivalent a.c. resistor of the rectifier connected to the loaded regulator	Ω
Link voltages		
V_{prim}	a.c. link input voltage	V
V_{sec}	a.c. link output voltage (= voltage across R_{load_2})	V
$V_{d.c.}$, $I_{d.c.}$	the rectified but non-regulated d.c. output of the driven inductive link (= voltage across and current through $R_{d.c.}$)	V, A
V_{out}, I_{out}	the rectified and regulated d.c. output of the driven inductive link (= voltage across and current through R_{out})	V, A
The primary tank		
ω_{tank}	resonance frequency of the primary tank, coupled to the secondary (i.e. amplitude resonance for saturated-class-C drivers and phase resonance for all other drivers).	rad/s
Q_{prim}	the quality factor of the primary tank, connected to the driver amplifier, but not coupled to the secondary (including coil losses, on-resistance of the driver transistors, resistive losses in capacitors,...)	-
Z_{eq}	the equivalent impedance of the secondary transformed to the primary (series R-L-C model for the primary coil)	Ω
R_{eq}	Z_{eq} that has become real at the phase-resonance frequency of the secondary tank (series R-L-C model for the primary coil)	Ω
$R^*_{s_1}$	series resistance of the primary coil coupled to the secondary circuit (R-L-C coil model) $R^*_{s_1} = R_{eq} + R_{s_1}$	Ω

Symbol	Description	Unit
$Q^*_{L_{s_1}}$	quality factor of the primary coil coupled to the secondary circuit (series R-L-C coil model)	-
	$Q^*_{L_{s_1}} = \frac{\omega L_{s_1}}{R^*_{s_1}} = \frac{\omega L_{s_1}}{R_{eq} + R_{s_1}}$	
$R^*_{L_1}$	series resistance of the primary coil coupled to the secondary circuit (R-L coil model)	Ω
$Q^*_{L_1}$	quality factor of the primary coil coupled to the secondary circuit (R-L coil model)	-
	$Q^*_{L_1} = \frac{\omega L_1}{R^*_{L_1}}$	
C_1	primary tank capacitor (class C and D drivers)	F
C_{1ser}	primary series capacitor in a class-E driver	F
C_{1par}	primary parallel capacitor in a class-E driver	F
C_{1res}	resonance capacitor in a (semi-) resonant class-E driver	F
Secondary tank		
ω_{resP}	phase-resonance frequency of the uncoupled secondary tank	rad/s
ω_{resA}	amplitude-resonance frequency of the uncoupled secondary tank (i.e. maximal voltage across R_{load_2})	rad/s
	$\omega_{resP} = \omega_{resA} \equiv \omega_{res}$ for a series-tuned secondary	
Q_{sec}	quality factor of the loaded, but uncoupled secondary tank	-
C_2	secondary tank capacitor	F
The inductive-link driver		
V_{cc}, I_{cc}	the d.c. supply voltage and current of the primary coil driver for the first-order-simplified driver model	V, I
R_{cc}	$R_{cc} \equiv \frac{V_{cc}}{I_{cc}}$	Ω
V^*_{cc}	real supply voltage that also accounts for the driver losses	V
	$V^*_{cc} \equiv \frac{V_{cc}}{\eta_{driver}}$	
v_{AE}, v_S	the voltage across the active element or switch of the primary coil driver	V
i_{AE}, i_S	the current through the active element or switch	A
p	the power output capability of the primary coil driver	-
	$p \equiv \frac{P_{link\ in}}{v_{AE_{\max}} i_{AE_{\max}}}$ or $p \equiv \frac{P_{link\ in}}{v_{S_{\max}} i_{S_{\max}}}$	
D	switch duty cycle	-
	$D \equiv \frac{t_{ON}}{t_{ON} + t_{OFF}}$	
t_R, t_F	switch rise and fall times	s
V_{Ssat}	switch saturation voltage (i.e. the extrapolation of the switch voltage-current curve to a zero current)	V
R_{ON}	switch-on resistance: $v_S = V_{Ssat} + R_{ON}\, i_S$	Ω
L_S	parasitic series inductance of the switch	H
P_{t_R}, P_{t_F}	switch dissipation due to non-zero rise and fall times	W
$P_{V_{Ssat}}$	switch dissipation due to non-zero saturation voltage	W
$P_{R_{ON}}$	power dissipation in the switch-on resistance	W
ζ	normalised slope of the switch voltage at turn-on ($\zeta = 0$ for class E and < 0 for saturated class C)	-
	$\zeta \equiv \frac{1}{\omega V_{cc}} \left.\frac{dv_S(t)}{dt}\right\vert_{turn-on}$	

Symbol	Description	unit
Link optimisation		
α	the ratio of R_{load_2} and the impedance of the capacitor C_2 $\alpha = \omega . C_2 . R_{load_2}$	-
X	abbreviation for $k^2 Q_{L_{S_1}} Q_{L_{S_2}}$	-
P_{in}	d.c. input power of the driven inductive link	W
$P_{link\ in}$	a.c. power into the coil set = $P_{in} . \eta_{driver}$	W
P_{sec}	total power delivered to the secondary = $P_{link\ in} . \eta_{primary}$	W
$P_{link\ out}$	useful a.c. power output of the coil set = $P_{link\ in} . \eta_{link}$	W
P_{out}	useful d.c. power output of the driven inductive link = $P_{in} . \eta$	W
$\eta_{primary}$	primary efficiency = ratio of the power transferred to the secondary circuit and the power put into the primary coil	-
$\eta_{secondary}$	secondary efficiency = ratio of the power dissipated in the load R_{load_2} and the total dissipation of the secondary circuit	-
η_{link}	link efficiency: $\eta_{link} = \eta_{primary} . \eta_{secondary}$	-
η_{rectif}	rectifier efficiency	
η_{regul}	regulator efficiency	-
η_{driver}	primary coil-driver efficiency $= \dfrac{P_{link\ out}}{P_{in}}$	
η	global efficiency of the driven inductive link $\eta = \eta_{driver} . \eta_{link} . \eta_{rectif} . \eta_{regul}$	-

Symbol	Description	Value
μ_0	permeability constant (in vacuum)	$4\pi\ 10^{-7}$ H/m
ε_0	permittivity constant (in vacuum)	$8.85419\ 10^{-12}$ F/m
c	speed of light (in vacuum)	$2.99792\ 10^{8}$ m/s

Abbr.	Description
a.c.	alternating current (which means a non-zero signal frequency)
ADC	analogue-to-digital converter
AM	amplitude modulation
ASK	amplitude shift keying
CW	continuous wave
DAC	digital-to-analogue converter
d.c.	direct current (which refers to a zero frequency signal)
EM	electromagnetic
EMI	electromagnetic interference
FM	frequency modulation
FSK	frequency shift keying
MO	master oscillator
MOPA	master-oscillator-power-amplifier configuration
PA	power amplifier
PCM	pulse code modulation
PDM	pulse duration modulation (a synonym for PWM)
PFM	pulse frequency modulation
PPM	pulse position modulation
PWM	pulse width modulation
r.f.	radio frequency
RMS	root mean square
RX	receiver
THD	total harmonic distorsion
TX	transmitter
VFO	voltage-controlled-frequency oscillator

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1

An introduction on telemetry

The ever-growing technical world surrounding us would be unimaginable without ***connections; pathways for information and/or power from one site to another***. The mutual distances range from micrometers for on-chip connections, to thousands of kilometres for satellite communications. Connections play a massive role in our world. Millions of kilometres of electrical wires and optical fibres are installed worldwide annually, in houses, cars and industrial plants, in electric and electronic appliances, as computer networks or power-grid lines.

The notion of connections as information carriers is not restricted to electrical or optical links. Shafts and pneumatic lines can be seen as their mechanical brethren. The concept of connective systems is not unique to the engineering world either. Our hormone system, for instance, is a beautiful example of a biochemical information network. This system is even matched to a power distribution network that transports adenosine-triphosphate (ATP) via the blood through the body and energises the muscle tissue. In fact, mankind itself is built on communication and connection. Every human interaction is an information exchange through speech, touch, writing, eye contact or body language.

This book studies magnetic induction as a connection medium for short-range telemetry. Telemetry is the engineering branch that occupies itself with wirelessly measuring parameters of objects. The remote telemetry unit is often small compared to the object being measured, in order not to obstruct the object's functionality. The present introduction first reviews common telemetry techniques to situate magnetic induction amongst its competitors and to outline the various data modulation methods available. Examples are often taken from the biomedical field because that's where telemetry originally started. This, nonetheless, doesn't restrict the methods to this field only. RFID (radio frequency identification) is an example of a booming market that shares many operation principles with telemetry.

K. Van Schuylenbergh and R. Puers, *Inductive Powering: Basic Theory and Application to Biomedical Systems*, Analog Circuits and Signal Processing, 1–39.

1.1 WIRELESS CONNECTIONS

The wireless approach is often dictated by boundary conditions between the connected sites that obstruct hard-wire cabling. Radio frequency broadcasting of radio and television are the most common examples, but the list of applications is virtually unlimited: satellite telephone, cellular telephone (GSM, CDMA, 3G, etc.), global positioning satellite (GPS), wireless monitoring of humans and animals (MACKAY, 1970, 1993; AMLANER and MACDONALD, 1980; JEUTTER, 1983a). The obstructions can be various:

- A large mutual distance that makes a cable connection uneconomical (e.g. telephone lines to remote regions) or even not feasible (e.g. connection to a satellite).
- A physical barrier that is too tough to penetrate or that looses its protective function once it perforated (e.g. in situ parameter monitoring in a chemical vessel or in the human body).
- An unknown location of the remote system (e.g. wildlife monitoring).
- The fact that the remote system is moving. Moving cables tend to fail after a while due to metal fatigue. Slip contacts on rotating drums or shafts usually show a poor reliability. The contacts are noisy and their resistance changes with time.

Wireless information transmission requires a local energy source at both sites. It may, however, not always be feasible to connect the remote unit to the mains, especially when it is small and or portable. Sometimes, even batteries can be unwanted because of their size, limited lifetime, toxicity e.g. when used in the human body, or temperature hazard when a circuit fails (e.g. in an explosive environment). This explains the need to have the base unit supply the remote system wirelessly in certain applications.

The wireless communication and/or energising methods are classified in three groups, based on how they physically establish the connection:

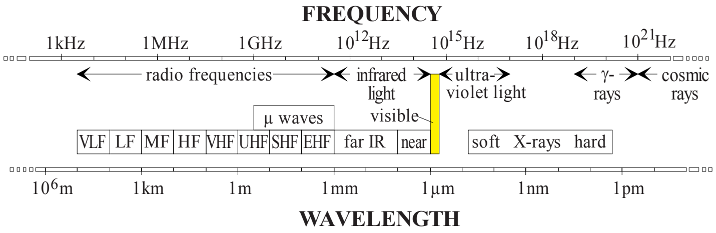
- Most wireless links are based on wave propagation:
  - Radio frequency (r.f.) links and optical links both rely on propagating electromagnetic waves (Figure 1.1). The radio frequencies are defined between 30 kHz and 300 GHz, whereas light ranges from far infrared (10^{12} Hz) to ultraviolet (10^{17} Hz).
  - The ultrasound links use acoustical waves with typical frequencies from some 10 kHz to some 10 MHz.

- A second group establishes the connection between both units, via electrical conduction through their intermediate medium.

- The third group couples the units with an alternating but non-radiating, quasi-stationary field:
  - The inductive links use magnetic coupling between two coils; one at each site. Typical frequencies are in the range of 1 kHz–100 MHz.
  - The capacitive links are established via an electrostatic field.[1]

---

[1] Capacitive links are only mentioned here for the sake of completeness as they only cover millimetre-range distances. However, the principle has been commercialised successfully in isolation amplifiers for the data transfer between the isolated circuit parts (TEXAS INSTRUMENTS, 2005).



The electromagnetic spectrum (*H*ALLIDAY *and* R*ESNICK, 1977*). *Figure 1.1.*

THE ELECTROMAGNETIC SPECTRUM (*SHANMUGAN, 1979*). *TABLE 1.1.*

Designation	**Frequency**	**Wavelength**	**Medium**	**Applications**
Very low frequency (VLF)	3–30 kHz	100–10 km	Wire pairs, long-wave radio	Audio, telephone, computer terminals, long range and undersea navigation, timing standards
Low frequency (LF)	30–300 kHz	10–1 km	Wire pairs, long-wave radio	Navigation, AM broadcasting (150–300 kHz), industrial (power line) communication
Medium frequency (MF)	0.3–3 MHz	1–0.1 km	Coaxial cable, long wave radio	AM broadcasting (540–1,600 kHz)
High frequency (HF)	3–30 MHz	100–10 m	Coax. cable, short-wave radio	AM broadcasting (6–30 MHz), CB and amateur radio, navigation and aviation
Very high frequency (VHF)	30–300 MHz	10–1 m	Coaxial cable, short-wave radio	Television band I (40–70 MHz) and III (175–225 MHz), FM broadcasting (88–108 MHz), air traffic control, amateur radio, police, fire brigade,...
Ultra high frequency (UHF)	0.3–3 GHz	1–0.1 m	Waveguides, μ-wave radio	Television band IV (450–600 MHz) and V (600–960 MHz), GSM, mobile radio, beamed transmission, radar
Super high frequency (SHF)	3–30 GHz	100–10 mm	Waveguides, μ-wave radio	Beamed transmission, satellite and space communication, radar
Extremely high frequency	30–300 GHz	10–1 mm	Waveguides, μ-wave radio	Navigation radar, radio astronomy,
Light (near IR .. UV)	10^{14}–10^{16} Hz	3 μm–30 nm	Optical fibres	Optical-fibre digital links

Different link principles are sometimes combined in a single application (HOF *et al.*, 1994): e.g. inductive powering together with an r.f. (FORSTER, 1986) or an infrared data link (MITAMURA *et al.,* 1990).

TABLE 1.2. A CLASSIFICATION OF MODULATION TECHNIQUES.

Continuous wave carrier modulation (CW):	• Amplitude-type modulation – Amplitude modulation (AM) – Double sideband modulation (DSB) – Single sideband modulation (SSB) • Frequency modulation (FM) • Phase modulation (PM) + Combinations by subcarrier modulation
Pulse modulation	• Analogue pulse modulation: – Pulse-position modulation (PPM) – Pulse-width modulation (PWM) – Pulse-frequency modulation (PFM) • Digital pulse-code modulation (PCM)

1.1.1 MODULATION

A steady radio frequency, acoustic or optical signal doesn't contain a lot of information. Directional antennas can estimate a transmitter's direction from the received signal intensity. Two separate direction-sensitive receivers can point the transmitter's position through triangulation. This makes steady transmission useful for, e.g. animal radio tracking, but that's about it! The transfer of "more intelligent" data requires modulation.

Modulation is the systematic variation of some attribute of a carrier waveform such as the amplitude, phase or frequency in accordance to the data signal. Despite the multitude of modulation techniques, two basic modulation types can be identified: the continuous wave (CW) or analogue modulation where the carrier attribute is modulated in a continuous fashion, and the pulse modulation where the carrier attribute is changed in a discrete abrupt manner (Table 1.2; BLACK, 1953; SCHWARTZ, 1970; SHANMUGAN, 1979; JEUTTER, 1983b).

1.1.1.1 Continuous wave carrier modulation

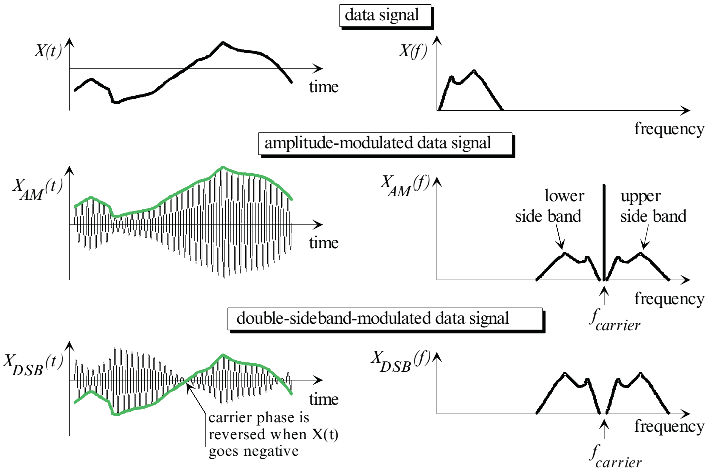
Three modulation types constitute the family of continuous wave-carrier modulations: amplitude modulation (with carrier or with suppressed carrier), frequency and phase modulation.

Amplitude modulation (AM) is a technique where the data signal directly controls the transmitter amplitude: when the data signal goes up, the transmitter output is increased accordingly (Figure 1.2):

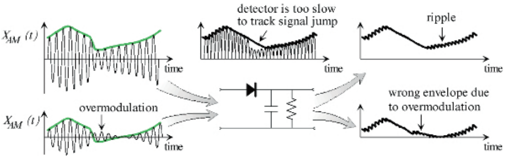
$$X_{AM}(t)=\left[1+m.X(t)\right].\cos(\omega_c.t)$$

The data signal $X(t)$ is supposed here to be normalised to a maximal amplitude of one. The modulation index m should be smaller than one to avoid envelope distortion. The carrier frequency ω_c is at least a decade larger than the highest data frequency so that a simple envelope detector can reconstruct the original data signal (Figure 1.3).

The carrier term in an AM signal contains no information, but represents a substantial component in the signal power. One can thus economise on transmitter power by suppressing this carrier. This kind of amplitude



Amplitude modulation and double-sideband modulation. *Figure 1.2.*



The principle of envelope detection for amplitude demodulation and the potential distortion causes. *Figure 1.3.*

modulation with suppressed carrier is called <u>double-sideband modulation</u> (DSB) and is accomplished by multiplying the data signal with a carrier[2]:

$$X_{DSB}(t) = X(t).\cos(\omega_c t)$$

The modulated data is recovered by multiplication with a coherent or synchronous carrier and low-pass filtering:

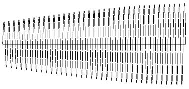
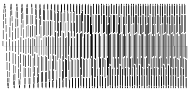
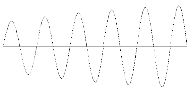
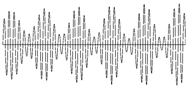
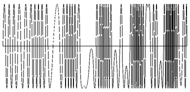
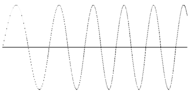
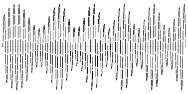
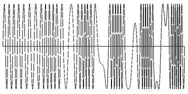
$$X_{re\,cov.}(t) = X_{DSB}(t).2\cos(\omega_c t) = X(t).2\cos^2(\omega_c t) = X(t) + \underbrace{X(t).\cos(2\omega_c t)}_{\text{rejected by low-pass filtering}}$$

The major problem in the data recovery is to have a coherent local carrier in the receiver. The original carrier is not contained in the DSB signal so it should be generated in the receiver. The higher circuit complexity involved in DSB and SSB rejects them for telemetry applications.

---

[2] DSB modulation still leaves some redundancy, as both side bands are instances of the same information. The next power-economising step is to suppress one of both sidebands to get single sideband modulation (SSB). This, however, further increases the circuit complexity and is therefore never used for small-sized telemetry units.

Table 1.3. *Composite amplitude - and frequency-modulation techniques.*

	submodulated signal	AM main carrier	FM main carrier
		AM	FM
without subcarrier	(= input signal)		
		AM-AM	AM-FM
AM sub-carrier			
		FM-AM	FM-FM
FM sub-carrier			

Neither AM, nor DSB or SSB can faithfully reproduce slowly varying data or absolute data values, as the reconstructed-data amplitude directly depends on the strength of the received modulated signal. Amplitude fading due to movements of mobile transmitters or changing atmospheric conditions is incorrectly interpreted as a declining data signal.

<u>Frequency modulation</u> (FM) and <u>phase modulation</u> (PM) are both insensitive to such amplitude changes, because they translate the data signal *X(t)* in a frequency shift or a carrier phase shift:

$$X_{FM}(t) = \cos\left[\left[1 + m.X(t)\right].\omega_c t\right] \quad \text{and} \quad X_{PM}(t) = \cos\left[\omega_c t + \pi.m.X(t)\right]$$

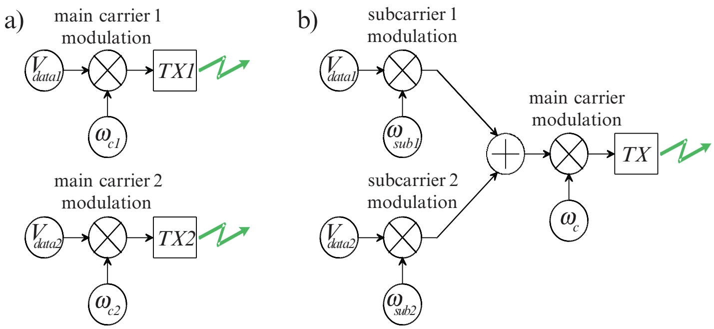
FM and PM are similar in functional form, except for the integration of the data signal in FM:

$$X_{FM}(t) = \cos\left(\omega_c t + m.X(t).\omega_c t\right) = \cos\left[\omega_c t + m.\omega_c \int_{-\infty}^{t} X(\tau).d\tau\right]$$

FM and PM signals are produced by a voltage-controlled-frequency oscillator (VFO) driven by the data signal or its time derivative. The data recovery is performed by the opposite sequence or by a phase-locked-loop circuit (PLL) (Shanmugan, 1979, pp. 289–297).

Simultaneous transmission of several data channels through the same medium is done by assigning one carrier frequency to each data channel (<u>frequency multiplexing</u>). As a consequence, increasing the number of output channels multiplies the number of transmitters and the corresponding energy consumption (Figure 1.4a). This becomes particularly problematic with small, battery-powered telemetry units. As the transmitter consumption dominates the overall power drain, this is the place to economise. Furthermore, the small size also limits the circuit complexity.

<u>Subcarrier modulation</u> is a technique to share one carrier frequency and transmitter with several data channels (Table 1.3; Figure 1.4b). Each data signal is first either amplitude or frequency modulated on an individual subcarrier. These contributions are then summed into a composite signal that is then amplitude or frequency modulated on an r.f. carrier (Klein



Two-channel data transmission, without (a) and with subcarrier modulation (b). *Figure 1.4.*

and DAVIS, 1976; PITSILLIDES *et al.*, 1992). Submodulation is thus also a form of frequency multiplexing, but it takes place on a lower, subcarrier, level.

It should be noted that subcarrier modulation pushes the transmitter frequencies to higher values. The subcarrier frequencies should be at least one decade larger than the highest signal frequency and well separated from each other to allow for easy channel filtering at the receiver. The carrier frequency is in its turn, another one or more decades higher than the maximal subcarrier frequency.

1.1.1.2 Analogue pulse modulation

Pulse modulation involves ***the discrete variation of a carrier attribute (amplitude, frequency or phase) at discrete time intervals corresponding to the modulation pulses.*** The modulation pulses actually represent a form of analogue sampling, which implies that the pulse frequency should be at least twice the upper signal frequency according to Nyquist's Theorem. The pulse frequency is limited at the upper side to about a twentieth of the carrier frequency so that each pulse takes at least about 10 carrier periods.

Three analogue-pulse modulations are distinguished (Table 1.4):

- Pulse-position modulation (PPM)) is a format of consecutive pulse pairs repeated at a fixed rate. Each pair represents a data sample: the first pulse is synchronised to a clock and the second pulse is then positioned such that the time between both is proportional to the momentary data value.
- Pulse-width modulation (PWM), also known as pulse-duration modulation (PDM), is another synchronous sampling method. Each sample value is coded as the width of a single pulse. The rising edges of the consecutive pulses are synchronised to a clock.
- Pulse-frequency modulation (PFM)[3] is an asynchronous format of single pulses. The time between consecutive pulses is proportional or inversely proportional to the momentary data value (both exist) (SANSEN and PUERS, 1984; SANSEN *et al.*, 1984a, b; PUERS and SANSEN, 1985). The sampling frequency thus varies with the data values.

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[3] Also often erroneously named pulse-position modulation.

TABLE 1.4. *ANALOGUE PULSE MODULATION.*

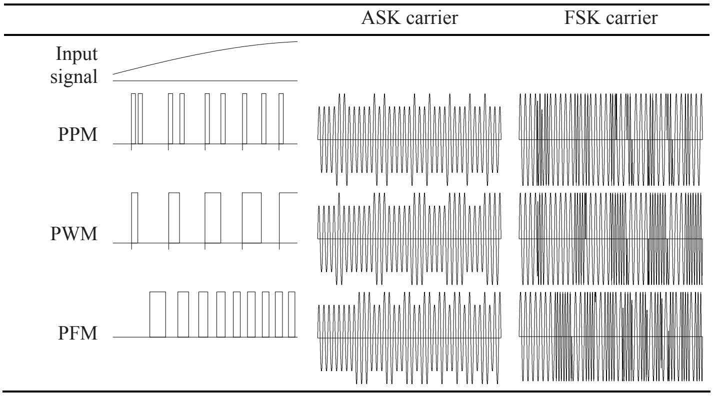
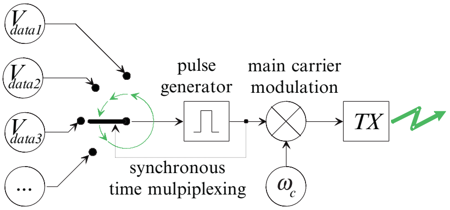


Figure 1.5. *Time multiplexing, synchronous with a pulse subcarrier.*



Amplitude-modulating the data pulses on an r.f. carrier is often referred to as amplitude shift keying (ASK) because the carrier amplitude is shifting between two distinct values (a pulse-on value and a pulse-off value). Pulse modulation on an FM or PM carrier is named frequency and phase shift keying (FSK and PSK) for the same reason.

The special kind of ASK where the transmitter is switched fully on and completely off ($m = 1$) by a pulse-modulated signal with a low duty-cycle, is very popular for battery-powered telemetry. The low duty cycle allows the transmitter to run on a higher peak power and hence a larger range for the same average power consumption (LIN and PILLAY, 1974).

Frequency multiplexing was pointed out before as a way to put multiple channels on a single carrier. Time division multiplexing is second way to share a single carrier frequency with multiple data channels. This method combines very well with pulse modulation. The transmitter input is switched from one data channel to the next one after a certain time, e.g. after each data pulse from the pulse-submodulator (Figure 1.5; EVANS, 1989). It may be useful to include a synchronisation pulse at the start of each channel scan to identify the first data channel and to synchronise the receiver with the switching pattern.