Göran Sandberg
Roger Ohayon
Editors

CISM
International Centre for Mechanical Sciences

Computational Aspects of Structural Acoustics and Vibration

CISM Courses and Lectures, vol. 505

SpringerWienNewYork

SpringerWienNewYork

CISM COURSES AND LECTURES

The series presents lecture notes, monographs, edited works and proceedings in the field of Mechanics, Engineering, Computer Science and Applied Mathematics.
Purpose of the series is to make known in the international scientific and technical community results obtained in some of the activities organized by CISM, the International Centre for Mechanical Sciences.

INTERNATIONAL CENTRE FOR MECHANICAL SCIENCES

COURSES AND LECTURES - No. 505

COMPUTATIONAL ASPECTS OF STRUCTURAL ACOUSTICS AND VIBRATION

EDITED BY

GÖRAN SANDBERG
LTH, LUND UNIVERSITY, SWEDEN

ROGER OHAYON
CNAM, PARIS, FRANCE

SpringerWienNewYork

This volume contains 113 illustrations

Printed in Italy
SPIN 12567607

All contributions have been typeset by the authors.

ISBN 978-3-211-89650-1 SpringerWienNewYork

PREFACE

Computational methods within structural acoustics, vibration and fluid-structure interaction are powerful tools for investigating acoustic and structural-acoustic problems in many sectors of industry; in the building industry regarding room acoustics, in the car industry and aeronautical industry for optimizing structural components with regard to vibrations characteristics etc. It is on the verge of becoming a common tool for noise characterization and design for optimizing structural properties and geometries in order to accomplish a desired acoustic environment.

This book is based on the course material given at CISM during the summer of 2006. The course started in the field of computational mechanics, and then moved into the field of formulations of multi-physics and multiscale and covered the following items

- *Structural dynamics,*
- *Various formulations for fluid-structure interaction, benefits and drawbacks, for*
  - *cavities,*
  - *external fluid,*
- *Modeling issues*
  - *validity of various formulations with regards to the physics,*
- *Source terms*
  - *e.g. vibrating equipment*
- *Application within the automotive industry and building acoustics.*

The book is addressed to graduate level, PhD students and young researchers interested in structural dynamics, vibrations and acoustics. It is also suitable for industrial researchers in mechanical, aeronautical and civil engineering with a professional interest in structural dynamics, vibrations and acoustics or involved in questions regarding noise characterization and reduction in building, car, plane, space, train, industries by means of computer simulations. We believe the book is of interest to researchers in applied mathematics wishing to learn about modern developments in the modelling of mechanical and civil engineering structures.

Göran Sandberg and Roger Ohayon

CONTENTS

Variational Formulations of Interior Structural-Acoustic Vibration Problems

Jean-François Deü and Walid Larbi and Roger Ohayon

Structural Mechanics and Coupled Systems Laboratory
Conservatoire National des Arts et Métiers
Case 353, 292 rue Saint-Martin, 75141 Paris Cedex 03, France

Abstract It is proposed to present appropriate variational formulations for linear vibration of elastic structure coupled with an internal acoustic fluid. Hybrid passive/active damping treatments will be investigated for noise and vibration reduction problems.

1 Introduction

It is proposed to present appropriate variational formulations for linear vibration of elastic structure coupled with an internal inviscid, homogeneous, compressible fluid (liquid or gas), gravity effects being discarded in the presence of a free surface. Hybrid passive/active damping treatments will be investigated for noise and vibration reduction problems. It should be noted that generally, active structural treatments (using for example piezoelectric smart materials) are effective in the low frequency range, while passive structural treatments (such as viscoelastic materials, porous insulation...) are effective for higher frequency domain.

In all the analyzed variational formulations, the structure will be described by a displacement field (the piezoelectric structure being described by an additional electric potential field). Concerning the fluid, instead of a description through a displacement field (for which we refer for example to (Bermúdez and Rodríguez, 1994; Park et al., 2001)) we will choose a scalar description through a pressure and/or a displacement potential field (Morand and Ohayon, 1995; Ohayon, 2004a,b).

Dissipative behavior is introduced through a fluid-structure wall damping modeling by local impedance connected with a viscoelastic Kelvin-Voigt type of constitutive equation. When taking into account dissipative structural-acoustic behavior through a local impedance constitutive equation, the problem becomes strongly frequency dependent (Kehr-Candille and Ohayon, 1992). In this presentation, we will use a simplify but rather general constitutive model of Kelvin-Voigt type through the introduction of a scalar interface variable which allows the problem to be reduced to a classical vibration damping problem (Deü et al., 2006; Larbi et al., 2006). This impedance model, though local, may represent relatively satisfactory porous medium (on a rigorous manner, a precise three-dimensional description of the porous medium at the fluid-structure interface would be necessary through Biot type approach (Davidsson and Sandberg, 2006)).

For piezoelectric structures (active treatments), structural-acoustic conservative formulation are extended in order to take into account electromechanical contributions. Here also, appropriate choice of variables has been investigated and leads to the introduction of the electric potential as an additional variable (Deü et al., 2008; Larbi et al., 2007).

For all the formulations, finite element discretization is discussed. Numerical results are then presented in order to illustrate the accuracy and versatility of the methodologies.

2 Conservative Structural-Acoustic Coupled Problem

Let us considered the free vibrations of an elastic structure completely filled with a homogenous, inviscid and compressible fluid, neglecting gravity effects. We establish in this section the variational formulation of the spectral problem and the corresponding matrix equations.

2.1 Local Equations

We consider an elastic structure occupying the domain Ω_S at the equilibrium. The structure is subjected to a prescribed displacement u_i^d on a part Γ_u and to surface force density F_i^d on the complementary part Γ_σ of its external boundary. The interior fluid domain is denoted by Ω_F and the fluid-structure interface by Σ (see Figure 1).

The structure is supposed to be relevant of classical linearized elasticity theory. Therefore, the stress tensor σ_{ij} is related to the linearized deformation tensor ε_{ij} by the constitutive law

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} \tag{1}$$

where c_{ijkl} are the coefficients of elasticity. We denote by ρ_S the mass density of the structure and n_i^S the unit normal external to Ω_S.

Since the compressible fluid is assumed to be inviscid, instead of describing its motion by a fluid displacement vector field, which requires an appropriate discretization of the fluid irrotationality constraint (Bermúdez and Rodríguez, 1994), we use the pressure scalar field p. Let us denote by

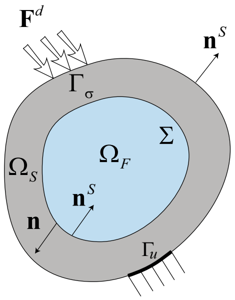

Figure 1. Structural-acoustic coupled system.

c_F the constant speed of sound in the fluid, by ρ_F the mass density of the fluid at equilibrium, and by n_i the unit normal external to Ω_F.

The local equations of the spectral structural-acoustic coupled problem described in Figure 1 are given by

$$\sigma_{ij,j}(u) + \omega^2 \rho_S u_i = 0 \qquad \text{in } \Omega_S \tag{2a}$$

$$\sigma_{ij}(u)\, n_j^S = 0 \qquad \text{on } \Gamma_\sigma \tag{2b}$$

$$u_i = 0 \qquad \text{on } \Gamma_u \tag{2c}$$

$$\sigma_{ij}(u)\, n_j^S = p\, n_i \qquad \text{on } \Sigma \tag{2d}$$

$$p_{,ii} + \frac{\omega^2}{c_F^2} p = 0 \qquad \text{in } \Omega_F \tag{3a}$$

$$p_{,i} n_i = \omega^2 \rho_F u_i\, n_i \qquad \text{on } \Sigma \tag{3b}$$

Equation (2a) corresponds to the classical elastodynamic equation expressed in terms of u; Equations (2b) and (2c) are the prescribed mechanical boundary conditions; Equation (2d) results from the action of pressure forces exerted by the fluid on the structure; Equation (3a) is the Helmholtz equation; and Equation (3b) expresses the continuity of the fluid and structural normal displacement components at the interface:

$$u_i n_i - \eta = 0 \qquad \text{on } \Sigma \tag{4}$$

where η represents the normal component of the fluid displacement field u_i^F $(\eta = u_i^F n_i)$.

In addition, the following constraint must be added

$$\rho_F c_F^2 \int_\Sigma u_i\, n_i \,\mathrm{d}s + \int_{\Omega_F} p \,\mathrm{d}v = 0, \tag{5}$$

in order to ensure that Equations (3a) and (3b) are well posed for $\omega = 0$ due to the scalar description of the fluid. From a physical point of view, this constraint ensures the satisfaction of mass conservation equation for $\omega = 0$. Moreover, if we consider the static response of the fluid to a prescribed deformation of the boundary Σ, the constraint (5) allows to find the static pressure:

$$p^s = -\frac{\rho_F c_F^2}{|\Omega_F|} \int_\Sigma u_i\, n_i \,\mathrm{d}s \tag{6}$$

Alternatively, considering the structure, the solution $\omega = 0$ is of course excluded due to Equation (2c) (the structure is fixed on Γ_u which eliminates any rigid body motion).

2.2 Variational Formulation

In order to obtain the variational formulation associated with the local equations of the coupled structural-acoustic system given in Equations (2)-(3), the test-function method is applied. We proceed in two steps, successively considering the equations relating to the structure (subject to fluid pressure actions) and the equations relating to the fluid (subject to a wall displacement).

First, we introduce the space C_u of sufficiently regular functions u_i defined in Ω_S and $C_u^* = \{u_i \in C_u \,|\, u_i = 0 \,\mathrm{on}\, \Gamma_u\}$. Multiplying Equation (2a) by any test-function $\delta u_i \in C_u^*$, then applying Green's formula, and finally taking Equations (2b) and (2d) into account, leads to

$$\int_{\Omega_S} c_{ijkl}\varepsilon_{kl}(u)\varepsilon_{ij}(\delta u) \,\mathrm{d}v - \omega^2 \rho_S \int_{\Omega_S} u_i\, \delta u_i \,\mathrm{d}v - \int_\Sigma p n_i\, \delta u_i \,\mathrm{d}s = 0 \tag{7}$$

Secondly, we consider the space C_p of sufficiently regular functions p defined in Ω_F. Multiplying Equation (3a) by any test-function $\delta p \in C_p$, applying Green's formula, and taking Equation (3b) into account, we obtain

$$\frac{1}{\rho_F} \int_{\Omega_F} p_{,i}\, \delta p_{,i} \,\mathrm{d}v - \frac{\omega^2}{\rho_F c_F^2} \int_{\Omega_F} p\, \delta p \,\mathrm{d}v - \omega^2 \int_\Sigma u_i\, n_i\, \delta p \,\mathrm{d}s = 0 \tag{8}$$

Finally, we have the following constraint for the static case

$$\rho_F c_F^2 \int_\Sigma u_i\, n_i \,\mathrm{d}s + \int_{\Omega_F} p \,\mathrm{d}v = 0 \tag{9}$$

Thus, the variational formulation of the structural-acoustic spectral problem consists in finding $\omega \in \mathbb{R}_+^*$ and $(u_i, p) \in (C_u^*, C_p)$, such that $\forall(\delta u_i, \delta p) \in (C_u^*, C_p)$, Equations (7), (8) and (9) are satisfied.

2.3 Finite Element Discretization

Let us introduce $\mathbf{U}$ and $\mathbf{P}$ corresponding to the vectors of nodal values of u_i and p respectively, and the matrices corresponding to the various bilinear forms involved in Equations (7)-(8) defined by

$$\int_{\Omega_S} c_{ijkl}\varepsilon_{kl}(u)\varepsilon_{ij}(\delta u)\,\mathrm{d}v \Rightarrow \delta\mathbf{U}^T\mathbf{K}_u\mathbf{U} \tag{10a}$$

$$\rho_S \int_{\Omega_S} u_i\,\delta u_i\,\mathrm{d}v \Rightarrow \delta\mathbf{U}^T\mathbf{M}_u\mathbf{U} \tag{10b}$$

$$\int_{\Sigma} p\,n_i\,\delta u_i\,\mathrm{d}s \Rightarrow \delta\mathbf{U}^T\mathbf{C}_{up}\mathbf{P} \tag{10c}$$

$$\int_{\Sigma} u_i\,n_i\,\delta p\,\mathrm{d}s \Rightarrow \delta\mathbf{P}^T\mathbf{C}_{up}^T\mathbf{U} \tag{10d}$$

$$\frac{1}{\rho_F}\int_{\Omega_F} p_{,i}\,\delta p_{,i}\,\mathrm{d}v \Rightarrow \delta\mathbf{P}^T\mathbf{K}_p\mathbf{P} \tag{10e}$$

$$\frac{1}{\rho_F c_F^2}\int_{\Omega_F} p\,\delta p\,\mathrm{d}v \Rightarrow \delta\mathbf{P}^T\mathbf{M}_p\mathbf{P} \tag{10f}$$

where $\mathbf{M}_u$ and $\mathbf{K}_u$ are the mass and stiffness matrices of the structure; $\mathbf{M}_p$ and $\mathbf{K}_p$ are the mass and stiffness matrices of the fluid; $\mathbf{C}_{up}$ is the fluid-structure coupled matrix.

Thus, the variational equations (7)-(8) for the structural-acoustic spectral problem can be written, in discretized form, as the following unsymmetric matrix system:

$$\begin{pmatrix} \mathbf{K}_u & -\mathbf{C}_{up} \\ \mathbf{0} & \mathbf{K}_p \end{pmatrix}\begin{pmatrix} \mathbf{U} \\ \mathbf{P} \end{pmatrix} - \omega^2 \begin{pmatrix} \mathbf{M}_u & \mathbf{0} \\ \mathbf{C}_{up}^T & \mathbf{M}_p \end{pmatrix}\begin{pmatrix} \mathbf{U} \\ \mathbf{P} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{11}$$

with the following relation which is related to the discretization of Equation (9)

$$\mathbf{L}_1^T\mathbf{U} + \mathbf{L}_2^T\mathbf{P} = 0 \tag{12}$$

This scalar relation is obtained by the discretization of the variational form of Equation (9) with constant test-functions. Therefore, $\mathbf{L}_1$ and $\mathbf{L}_2$ are obtained from Equations (10d) and (10f) with constant $\delta\mathbf{P}$

$$\mathbf{L}_1 = \delta\mathbf{P}^T\mathbf{C}_{up}^T$$

$$\mathbf{L}_2 = \delta\mathbf{P}^T\mathbf{M}_p$$

From a computational point of view, this formulation has the advantage of only introducing one unknown per node to describe the fluid. On the other hand, the matrix system is non-symmetric needed to use particular eigenvalue solvers. In order to avoid this drawback, various symmetric formulations have been derived. The symmetrization is carried out at the continuum level through the description of the fluid by two scalar fields, namely pressure p and displacement potential φ ($u_i^F = \varphi_{,i}$). This displacement potential is defined up to an additive constant. It should be noted that one of the scalar variables can be eliminated through generalized added mass or added stiffness operators (Morand and Ohayon, 1995).

3 Dissipative Structural-Acoustic Coupled Problem

In this section, we propose to investigate the effect of introducing a thin layer of absorbing material (for example porous insulated material) at the fluid-structure interface in order to damp the elastoacoustic energy. In the present analysis, we suppose that this physical interface can be modeled by a massless geometric surface. Therefore, this interface will be described by a particular constitutive law through the introduction of a dissipative wall acoustic impedance $Z(\omega)$ (Ohayon and Soize, 1998). As a consequence, the equation (4) must be replaced by:

$$p = i\omega Z(\omega)(u_i n_i - \eta) \tag{14}$$

which quantify the normal displacement discontinuity at the interface.

In many situations, it has been verified that $Z(\omega)$ can be approximated by a viscoelastic Kelvin-Voigt model (see Figure 2), i.e. by the sum of a constant real part and an imaginary part inversely proportional to the frequency: $Z(\omega) = d^I + ik^I/\omega$. The constant parameters k^I and d^I are associated respectively to the elastic and viscous contributions of the absorbing layer. Equation (14) then writes:

$$p = -(k^I - i\omega d^I)(u_i n_i - \eta) \tag{15}$$

We establish here (i) the variational formulation of the structural-acoustic problem with damping interface using a Kelvin-Voigt model and (ii) the corresponding matrix equations resulting for instance from a finite element discretization.

3.1 Local Equations in Terms of (u_i, η, p)

The local equations of the spectral coupled problem with damping interface are expressed in terms of the structural displacement field u_i, the normal fluid-structure interface displacement η, and the fluid pressure p

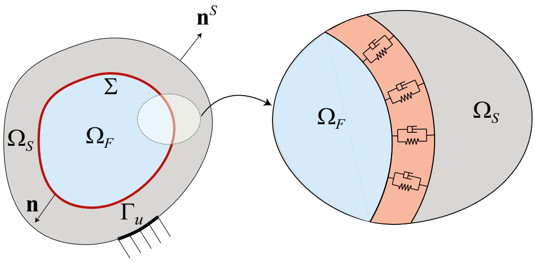

Figure 2. Structural-acoustic coupled system with damping interface.

$$\sigma_{ij,j}(u) + \omega^2 \rho_S u_i = 0 \qquad \text{in } \Omega_S \quad (16a)$$
$$\sigma_{ij}(u)\, n_j^S = 0 \qquad \text{on } \Gamma_\sigma \quad (16b)$$
$$u_i = 0 \qquad \text{on } \Gamma_u \quad (16c)$$
$$\sigma_{ij}(u)\, n_j^S = -\left[k^I(u_i n_i - \eta) - i\omega d^I(u_i n_i - \eta)\right] n_i \qquad \text{on } \Sigma \quad (16d)$$

$$p_{,ii} + \frac{\omega^2}{c_F^2} p = 0 \qquad \text{in } \Omega_F \quad (17a)$$
$$p_{,i} n_i = \omega^2 \rho_F \eta \qquad \text{on } \Sigma \quad (17b)$$

$$-p = k^I(u_i n_i - \eta) - i\omega d^I(u_i n_i - \eta) \qquad \text{on } \Sigma \quad (18)$$

Equation (16d) represents the force equilibrium at the interface; Equation (17b) represents the continuity condition between the fluid and the damping interface; and Equation (18) expresses the constitutive law of the interface: the first term is proportional to the normal displacement component and accounts the elastic behavior of the interface material, the second one is proportional to the normal velocity and models the viscous damping.

A relationship similar to Equation (9) must be added to ensure the well position of the fluid part of the problem for $\omega = 0$. For sake of simplicity, we do not introduce this constraint in the sequel (this analysis is the subject of current investigations).

3.2 Variational Formulation

The variational formulation of the structural-acoustic spectral problem with damping interface is obtained using the test-function method.

Multiplying Equation (16a) by any test-function $\delta u_i \in C_u^*$, then applying Green's formula, and finally taking Equations (16b), (16d) into account, leads to

$$\begin{aligned}
&\int_{\Omega_S} c_{ijkl}\varepsilon_{kl}(u)\varepsilon_{ij}(\delta u)\,\mathrm{d}v - \omega^2\rho_S\int_{\Omega_S} u_i\,\delta u_i\,\mathrm{d}v - k^I\int_\Sigma \eta n_i\delta u_i\,\mathrm{d}s\\
&+ k^I\int_\Sigma (u_i n_i)n_i\delta u_i\,\mathrm{d}s + i\omega d^I\int_\Sigma \eta n_i\delta u_i\,\mathrm{d}s - i\omega d^I\int_\Sigma (u_i n_i)n_i\delta u_i\,\mathrm{d}s = 0
\end{aligned} \tag{19}$$

Similarly, multiplying Equation (17a) by any test-function $\delta p \in C_p$, applying Green's formula, and taking Equation (17b) into account, we obtain

$$\frac{1}{\rho_F}\int_{\Omega_F} p_{,i}\,\delta p_{,i}\,\mathrm{d}v - \frac{\omega^2}{\rho_F c_F^2}\int_{\Omega_F} p\,\delta p\,\mathrm{d}v - \omega^2\int_\Sigma \eta\,\delta p\,\mathrm{d}s = 0 \tag{20}$$

Finally, we consider the space C_η of sufficiently regular functions η defined on Σ. Multiplying Equation (18) by $\delta\eta \in C_\eta$, we have:

$$\begin{aligned}
&k^I\int_\Sigma \eta\,\delta\eta\,\mathrm{d}s - k^I\int_\Sigma u_i n_i\,\delta\eta\,\mathrm{d}s - i\omega d^I\int_\Sigma \eta\,\delta\eta\,\mathrm{d}s + i\omega d^I\int_\Sigma u_i n_i\,\delta\eta\,\mathrm{d}s\\
&\qquad - \int_\Sigma p\,\delta\eta\,\mathrm{d}s = 0
\end{aligned} \tag{21}$$

Thus, the variational formulation of the structural-acoustic spectral problem with damping interface consists in finding $\omega \in \mathbb{C}_+^*$ and $(u_i, \eta, p) \in (C_u^*, C_\eta, C_p)$, such that $\forall(\delta u_i, \delta\eta, \delta p) \in (C_u^*, C_\eta, C_p)$, Equations (19)-(21) are satisfied.

In principle, one should use complex test-functions and sesquilinear products involving $(\overline{\delta u_i}, \overline{\delta p}, \overline{\delta\eta})$ complex conjugate quantities of $(\delta u_i, \delta p, \delta\eta)$. As the next subsection deals with finite element discretization, for sake of brevity, real test functions has been introduced but of course ω is complex.

3.3 Finite Element Discretization

Let us introduce **U**, **H** and **P** corresponding to the vectors of nodal values of u_i, η and p respectively, and the following submatrices corresponding to

Equations (19)-(21)

$$\int_{\Sigma} (u_i n_i) n_i \delta u_i \, \mathrm{d}s \Rightarrow \delta \mathbf{U}^T \mathbf{D}_u \mathbf{U} \tag{22a}$$

$$\int_{\Sigma} \eta \, n_i \delta u_i \, \mathrm{d}s \Rightarrow \delta \mathbf{U}^T \mathbf{C}_{u\eta} \mathbf{H} \tag{22b}$$

$$\int_{\Sigma} u_i n_i \, \delta\eta \, \mathrm{d}s \Rightarrow \delta \mathbf{H}^T \mathbf{C}_{u\eta}^T \mathbf{U} \tag{22c}$$

$$\int_{\Sigma} \eta \, \delta\eta \, \mathrm{d}s \Rightarrow \delta \mathbf{H}^T \mathbf{D}_{\eta} \mathbf{H} \tag{22d}$$

$$\int_{\Sigma} p \, \delta\eta \, \mathrm{d}s \Rightarrow \delta \mathbf{H}^T \mathbf{C}_{\eta p} \mathbf{P} \tag{22e}$$

$$\int_{\Sigma} \eta \, \delta p \, \mathrm{d}s \Rightarrow \delta \mathbf{P}^T \mathbf{C}_{\eta p}^T \mathbf{H} \tag{22f}$$

Using Equations (10) and (22), the variational Equations (19)-(21) leads to the following unsymmetric spectral matrix system:

$$\begin{pmatrix} \mathbf{K}_u + k^I \mathbf{D}_u & -k^I \mathbf{C}_{u\eta} & \mathbf{0} \\ -k^I \mathbf{C}_{u\eta}^T & k^I \mathbf{D}_{\eta} & -\mathbf{C}_{\eta p} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_p \end{pmatrix} \begin{pmatrix} \mathbf{U} \\ \mathbf{H} \\ \mathbf{P} \end{pmatrix} - i\omega \begin{pmatrix} d^I \mathbf{D}_u & -d^I \mathbf{C}_{u\eta} & \mathbf{0} \\ -d^I \mathbf{C}_{u\eta}^T & d^I \mathbf{D}_{\eta} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{U} \\ \mathbf{H} \\ \mathbf{P} \end{pmatrix} - \omega^2 \begin{pmatrix} \mathbf{M}_u & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{\eta p}^T & \mathbf{M}_p \end{pmatrix} \begin{pmatrix} \mathbf{U} \\ \mathbf{H} \\ \mathbf{P} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{23}$$

Remarks.

- It can be shown that the conservative (u_i, p) fluid-structure system described in Section 2 is obtained by setting $d^I = 0$ then $k^I \to \infty$ in the previous equations.
- In order to take into account possible mass effect of the interface (surface smeared-mass), one has to choose the impedance under the form $Z(\omega) = d^I - i(m^I \omega - k^I/\omega)$.
- Of course, as mentioned in Section 2, symmetrization of Equation (23) can be carried out by using an additional scalar variable to describe

the fluid (φ), leading, after elimination procedures, to added mass and added stiffness operators.

4 Piezoelectric Structural-Acoustic Coupled Problem

It is well known that noise reduction techniques using absorbing materials are quite effective at relatively medium and high frequency range. In the low frequency range, active techniques using piezoelectric materials are found to be an attractive alternative or complementary tool. In this case, sensor and actuator piezoelectric patches are surface-mounted or embedded in the structure. These patches are capable of self-sensing and self-actuation for active vibration and noise control. In this context, we present in this section variational and finite element formulations for the free vibration analysis of piezoelectric structural-acoustic coupled problems.

4.1 Local Equations of the Coupled Electro-Mechanical Problem

The local equations of the piezoelectric structural-acoustic problem are given by

$$\sigma_{ij,j} + \omega^2 \rho_S u_i = 0 \qquad \text{in } \Omega_S \tag{24a}$$

$$\sigma_{ij}\, n_j^S = 0 \qquad \text{on } \Gamma_\sigma \tag{24b}$$

$$u_i = 0 \qquad \text{on } \Gamma_u \tag{24c}$$

$$\sigma_{ij}\, n_j^S = p\, n_i \qquad \text{on } \Sigma \tag{24d}$$

$$D_{i,i} = 0 \qquad \text{in } \Omega_S \tag{25a}$$

$$D_i\, n_i^S = 0 \qquad \text{on } \Gamma_D \tag{25b}$$

$$\psi = 0 \qquad \text{on } \Gamma_\psi \tag{25c}$$

$$p_{,ii} + \frac{\omega^2}{c_F^2} p = 0 \qquad \text{in } \Omega_F \tag{26a}$$

$$p_{,i} n_i = \omega^2 \rho_F u_i\, n_i \qquad \text{on } \Sigma \tag{26b}$$

Equation (25a) corresponds to the Gauss law of electrostatics in absence electric of charge density. We recall that D_i denotes the electric displacement vector. Equations (25b) and (25c) are the electric boundary conditions. These electric boundary conditions are defined by a prescribed electric potential ψ^d on Γ_ψ and a surface density of electric charge Q^d on the remaining part Γ_D.

The stress tensor σ_{ij} and electric displacement D_i are related to the linear strain tensor ε_{kl} and electric field E_k through the converse and direct linear piezoelectric constitutive equations

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl}(u) - e_{kij}E_k(\psi) \tag{27}$$

$$D_i = e_{ikl}\varepsilon_{kl}(u) + \epsilon_{ik}E_k(\psi) \tag{28}$$

where c_{ijkl}, e_{kij} and ϵ_{ik} denote elastic, piezoelectric and dielectric material constants.

Moreover, we have the following gradient relations between the linearized strain tensor ε_{kl} and the displacement u_k, and between the electric field vector E_k and the electric potential ψ:

$$\varepsilon_{kl} = \frac{1}{2}\left(u_{k,l} + u_{l,k}\right) \tag{29}$$

$$E_k = -\psi_{,k} \tag{30}$$

We can note from the constitutive Equations (27) and (28) and from the gradient relations (29) and (30) that the stress tensor σ_{ij} and the electric displacement vector D_i depend on the variables u_k and ψ:

$$\sigma_{ij} = \sigma_{ij}(u_k, \psi) \tag{31a}$$

$$D_i = D_i(u_k, \psi) \tag{31b}$$

For a detailed derivation of these classical equations, we refer the reader, for example, to (Tiersten, 1969) for piezoelectric aspects and to (Morand and Ohayon, 1995) for fluid-structure aspects.

4.2 Variational Formulation

The local equations of Section 4.1 are expressed in terms of the chosen unknown fields of the piezoelectric structural-acoustic boundary value problem, i.e. the structural mechanical displacement u_i, the electric potential in the structure ψ, and the fluid pressure p.

In order to obtain the variational formulation associated with the local equations of the coupled fluid/piezoelectric-structure system given in Equations (24)-(26), the test-function method is applied. We proceed in three steps, successively considering the equations relating to the structure (subject to fluid pressure actions), the electric charge equation for a dielectric medium, and the equations relating to the fluid (subject to a wall displacement).

First, multiplying Equation (24a) by any test-function $\delta u_i \in C_u^*$, integrating over Ω_S, then applying Green's formula, and finally taking Equations (24b), (24d), (27) and (29) into account, leads to

$$\int_{\Omega_S} c_{ijkl}\varepsilon_{kl}(u)\varepsilon_{ij}(\delta u)\,\mathrm{d}v - \int_{\Omega_S} e_{kij}E_k(\psi)\varepsilon_{ij}(\delta u)\,\mathrm{d}v - \int_{\Sigma} pn_i\,\delta u_i\,\mathrm{d}s - \omega^2\rho_S\int_{\Omega_S} u_i\,\delta u_i\,\mathrm{d}v = 0 \quad (32)$$

Secondly, we consider the space C_ψ of sufficiently regular functions ψ in Ω_S and $C_\psi^* = \{\psi \in C_\psi \,|\, \psi = 0 \,\mathrm{on}\, \Gamma_\psi\}$. Multiplying Equation (25a) by any test-function $\delta\psi \in C_\psi^*$, integrating over Ω_S and finally taking Equations (25b), (28) and (30) into account, we have

$$\int_{\Omega_S} e_{ikl}\varepsilon_{kl}(u)E_i(\delta\psi)\,\mathrm{d}v + \int_{\Omega_S} \epsilon_{ik}E_k(\psi)E_i(\delta\psi)\,\mathrm{d}v = 0 \quad (33)$$

Finally, Multiplying Equation (26a) by any test-function $\delta p \in C_p$, applying Green's formula, and taking Equation (26b) into account, we obtain

$$\frac{1}{\rho_F}\int_{\Omega_F} p_{,i}\,\delta p_{,i}\,\mathrm{d}v - \frac{\omega^2}{\rho_F c^2}\int_{\Omega_F} p\,\delta p\,\mathrm{d}v - \omega^2\int_{\Sigma} u_i\,n_i\,\delta p\,\mathrm{d}s = 0 \quad (34)$$

Thus, the variational formulation of the fluid/piezoelectric-structure spectral problem consists in finding $\omega \in \mathbb{R}^+$ and $(u_i, \psi, p) \in (C_u^*, C_\psi^*, C_p)$, such that $\forall(\delta u_i, \delta\psi, \delta p) \in (C_u^*, C_\psi^*, C_p)$, Equations (32) and (34) are verified.

4.3 Finite Element Discretization

The discretization of the preceding variational formulation leads to the following matrix equation

$$\begin{pmatrix} \mathbf{K}_u & -\mathbf{C}_{u\psi} & -\mathbf{C}_{up} \\ \mathbf{C}_{u\psi}^T & \mathbf{K}_\psi & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_p \end{pmatrix} \begin{pmatrix} \mathbf{U} \\ \mathbf{\Psi} \\ \mathbf{P} \end{pmatrix} - \omega^2 \begin{pmatrix} \mathbf{M}_u & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{C}_{up}^T & \mathbf{0} & \mathbf{M}_p \end{pmatrix} \begin{pmatrix} \mathbf{U} \\ \mathbf{\Psi} \\ \mathbf{P} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \quad (35)$$

where $\mathbf{U}$, $\mathbf{\Psi}$ and $\mathbf{P}$ are the vectors of nodal values of u_i, ψ and p respectively, and where the not yet defined submatrices of Equations (32)-(34) are given by

$$\int_{\Omega_S} e_{kij} E_k(\psi) \varepsilon_{ij}(\delta u)\, \mathrm{d}v \Rightarrow \delta \mathbf{U}^T \mathbf{C}_{u\psi} \mathbf{\Psi} \tag{36a}$$

$$\int_{\Omega_S} e_{ikl} \varepsilon_{kl}(u) E_i(\delta\psi)\, \mathrm{d}v \Rightarrow \delta \mathbf{\Psi}^T \mathbf{C}_{u\psi}^T \mathbf{U} \tag{36b}$$

$$\int_{\Omega_S} \epsilon_{ik} E_k(\psi) E_i(\delta\psi)\, \mathrm{d}v \Rightarrow \delta \mathbf{\Psi}^T \mathbf{K}_{\psi} \mathbf{\Psi} \tag{36c}$$

$\mathbf{K}_\psi$ (resp. $\mathbf{C}_{u\psi}$) represents the electric stiffness (resp. the electric mechanical coupled stiffness) matrix.

Remarks.

- $\mathbf{K}_\psi$ is invertible because $\psi = 0$ on Γ_ψ. The case of $\Gamma_\psi = \emptyset$ will be the subject of further investigations.

- The elimination of electric potential degrees of freedom $\mathbf{\Psi}$ using the second row of Equation (35), leads to

$$\begin{pmatrix} \mathbf{K}_u + \mathbf{K}^A & -\mathbf{C}_{up} \\ \mathbf{0} & \mathbf{K}_p \end{pmatrix} \begin{pmatrix} \mathbf{U} \\ \mathbf{p} \end{pmatrix} - \omega^2 \begin{pmatrix} \mathbf{M}_u & \mathbf{0} \\ \mathbf{C}_{up}^T & \mathbf{M}_p \end{pmatrix} \begin{pmatrix} \mathbf{U} \\ \mathbf{P} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{37}$$

  where the added-stiffness matrix $\mathbf{K}^A = \mathbf{C}_{u\psi} \mathbf{K}_\psi^{-1} \mathbf{C}_{u\psi}^T$ is due to the electromechanical coupling.

- When the piezoelectric coupling constants are set to zero, the added stiffness matrix $\mathbf{K}^A = \mathbf{0}$ and we obtained the unsymmetric (u_i, p) fluid-structure system involved in Section 2.

- As in the previous section, a symmetric formulation can be established by introducing the fluid displacement potential as additional variable (Deü et al., 2008).

5 Numerical Examples

We present in this section some finite element results, obtained with the previous formulations, for the free vibration analysis of interior structural-acoustic systems. First, a three-dimensional vibroacoustic problem with damping interface is analyzed. The second example concerns the free vibration analysis of a piezoelectric cylinder filled with fluid.

5.1 Vibration Analysis of a Plate/Acoustic Cavity with Damping Interface

We consider in this first example the spectral problem of a 3D rectangular acoustic cavity of size $A = 0.6$ m, $B = 0.5$ m and $C = 0.4$ m (see Figure 2a) completely filled with air ($\rho_F = 1$ kg/m^3, $c_F = 340$ m/s). One wall of the cavity is a flexible plate of thickness 6 mm clamped by its whole boundary and covered with a thin layer of absorbing material. The other walls are considered perfectly rigid. The mechanical parameters of the plate are: density $\rho_S = 7700$ Kg/m^3, Young's modulus $E = 1.44 \times 10^{11}$ Pa and Poisson ratio $\nu = 0.35$. The absorbing material, which is considered massless in this example, has two parameters: $k^I = 5 \times 10^6$ N/m^3 and $\mathrm{d}^I = 50$ Ns/m^3. These parameters are average impedance coefficients corresponding to a typical acoustic insulating fabric (a Johns Manville glass wool of thickness 1 inch) in the frequency range (50–500 Hz) (see Figure 2b).

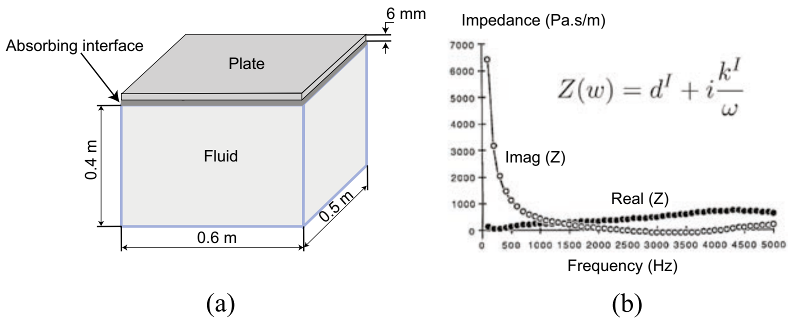

Figure 3. Plate/acoustic cavity system: geometrical data and acoustic impedance.

Table 1 presents the first four eigenfrequencies (in Hz) with uniform meshes (hexagonal element) and with increasing number of degrees of freedom for the 3D acoustic cavity with and without damping interface. The first and second columns present the frequencies of the rigid cavity computed from a pressure formulation and compared to the exact solution $f_{mnk} = (c_F/2)\sqrt{m^2/A^2 + n^2/B^2 + k^2/C^2}$. The three other columns correspond to the complex frequencies of the damping cavity computed from the proposed formulation and compared to exact solution (last column) given in (Bermúdez et al., 2001). A good agreement between exact and computed values can be observed even for the coarse mesh. In this example, the imaginary part of the frequencies comes from dashpot dissipation. Moreover, the difference between the real part (damped case) and the real value (undamped case) of the frequencies is due to the spring effect. Figures 4 and 5 show the pressure field in the acoustic cavity for the damped and undamped cases.

Table 1. Frequencies (Hz) of a 3D rigid acoustic cavity with an absorbing wall.

undamped		damped		
(4096 dof)	exact	(1452 dof)	(4352 dof)	exact*
283.85	$f_{100} = 283.33$	275.98-0.15i	275.35-0.15i	274.85-0.15i
340.62	$f_{010} = 340.00$	330.81-0.23i	330.06-0.23i	329.46-0.23i
425.78	$f_{001} = 425.00$	403.32- 0.55i	402.59-0.54i	402.00-0.54i
443.39	$f_{110} = 442.58$	429.45-0.46i	428.48-0.46i	427.71-0.46i

$*$: (Bermúdez et al., 2001)

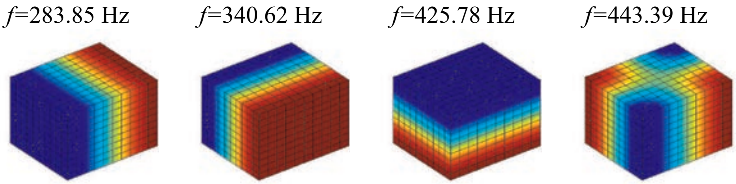

Figure 4. First four acoustic modes for the 3D acoustic rigid cavity.

Table 5.1 presents the eigenfrequencies in four cases: (i) 3D rigid acoustic cavity; (ii) clamped plate; (iii) plate/acoustic cavity coupled system without damping; and (iv) plate/acoustic cavity coupled system with damping interface. Note that in the third and forth cases, our results are compared to those given in (Bermúdez et al., 2001).

Firstly, it must be noted that the present results are in good agreement with those obtained in (Bermúdez et al., 2001) with a displacement formulation for both domains (fluid and structure). As shown in this table, modes A, C and E correspond to the first three vibration modes of the structure (lower than 400 Hz) and the four others (B, D, F and G) are the first four acoustic modes. In the damped case, notice that although the real parts of the frequencies corresponding to the structure modes remain practically unchanged, those associated with the fluid modes decrease between 2 and 10 %. The imaginary parts of the frequencies are almost zero for the structure modes, which means that they are only very slightly damped. As expected, the imaginary parts of the fluid modes are higher. Thus, the damping is

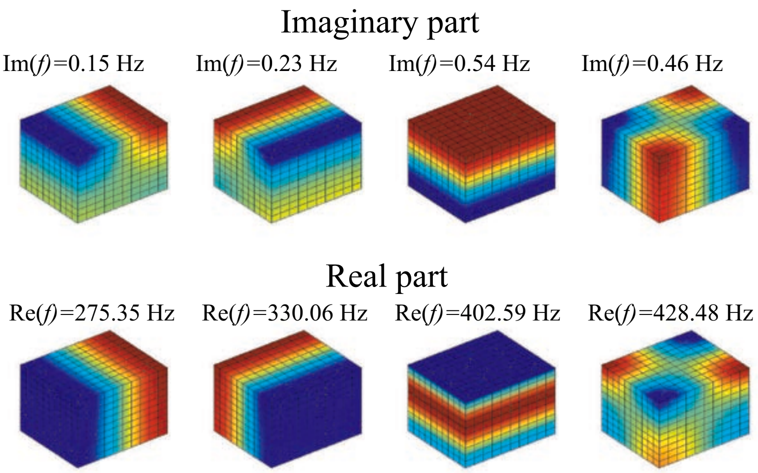

Figure 5. First four acoustic modes for the 3D acoustic cavity with an absorbing wall (real and imaginary parts).

Table 2. Computed frequencies (Hz) of the structural-acoustic coupled system.

Mode	undamped				damped	
	F.(i) 4096 dof	S. (ii) 980 dof	FSI*	FSI (iii) 5076 dof	FSI*	FSI (iv) 5332 dof
A	-	158.13	156.61	158.18	156.91-0.00i	158.18-0.00i
B	283.85	-	280.90	281.91	273.43-0.17i	275.30-0.18i
C	-	290.24	294.37	291.95	294.07-0.01i	291.75-0.00i
D	340.62	-	338.01	339.93	326.64-0.31i	330.43-0.22i
E	-	362.83	375.80	363.19	375.97-0.01i	375.80-0.01i
F	425.78	-	422.97	425.89	394.04-1.30i	403.49-0.55i
G	443.39	-	441.91	443.07	417.79-1.72i	429.21-0.46i

$*$: (Bermúdez et al., 2001)

stronger for the acoustic modes. For illustration purposes, Figure 6 shows the deformed plate and the pressure field for the first vibration mode (A) in the coupled case.

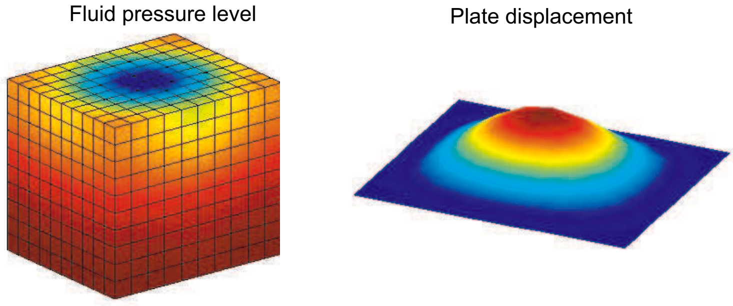

Figure 6. First coupled mode without damping interface: fluid pressure level and plate total displacement.

5.2 Free Vibration of a Piezoelectric Cylindrical Shell Filled with Fluid

This second example concerns the free vibration analysis of a simply-supported piezoelectric cylindrical shell filled with air. Results are computed via a specific Matlab program developed for axisymmetric geometries. The structure is discretized using a laminated piezoelectric conical shell element. This two-node element is based on the Kirchhoff-Love theory and combines an equivalent single layer approach for the mechanical behavior with a layer-wise representation of the electric potential in the thickness direction. The fluid domain is discretized with quadrilateral axisymmetric elements. Moreover, a semi-analytical procedure combining the finite element method and Fourier series expansion in the circumferential direction is used. For more details, the reader is referred to (Larbi et al., 2007).

In this example, the geometrical properties are $L = 1$ m, $R = 0.2$ m and $h = 0.001$ m (Figure 7). Moreover, the piezoelectric material is the PZT-5H whose properties are given in (Deü and Larbi, 2006) and the considered fluid has a mass density $\rho_F = 1.21$ kg/m^3 and a speed of sound $c_F = 343$ m/s.

Table 3 presents the eigenfrequencies (given for the two first circumferential harmonics) of the coupled system computed by our finite element approach and those given by an exact three-dimensional solution proposed by

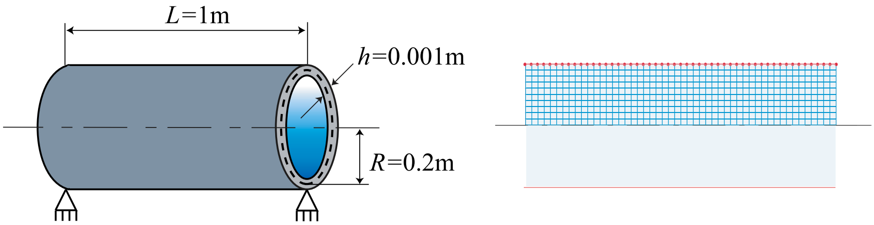

Figure 7. Geometrical data and meshes of the piezoelectric shell filled with fluid.

the authors (Deü and Larbi, 2006). This exact solution is based on a mixed state-space approach previously developed for the free-vibration analysis of laminated piezoelectric plates actuated by transverse shear mechanisms.

Table 3. Frequencies (Hz) of a piezoelectric cylindrical shell filled with a compressible fluid: Comparison with an exact state space solution (Deü and Larbi, 2006).

	Short circuited			Open circuited		
n	present	exact	error %	present	exact	error %
1	430.9	431.9	0.23	455.9	457.3	0.31
	1081.3	1080.7	-0.06	1140.5	1139.9	-0.05
	1393.6	1390.1	-0.25	1393.6	1390.1	-0.25
	1585.9	1583.9	-0.13	1665.6	1662.9	-0.16
	1848.4	1843.7	-0.25	1918.6	1915.5	-0.16
2	178.2	178.6	0.22	192.4	192.8	0.21
	561.0	561.1	0.02	602.7	602.9	0.03
	1006.9	1014.1	0.71	1054.3	1055.8	0.14
	1300.0	1297.0	-0.23	1393.3	1390.1	-0.23
	1541.2	1536.1	-0.33	1655.3	1650.5	-0.29

As it can be observed from this table, there is a very good agreement between finite element and exact solutions for different electric boundary conditions corresponding to short-circuited ($\psi^d = 0$ on $\partial\Omega_S$) or open-circuited ($Q^d = 0$ on $\partial\Omega_S$) configurations. These boundary conditions are prescribed on the inner and outer surface of the piezoelectric shell. Note that the error committed by the finite element approximation is lower than 1 %, validating the finite element electro-mechanical-acoustic formulation.

We can observe that the frequency of the third mode for $n = 1$ (1393.1 Hz) is not influenced by the electric boundary conditions. This is due to the

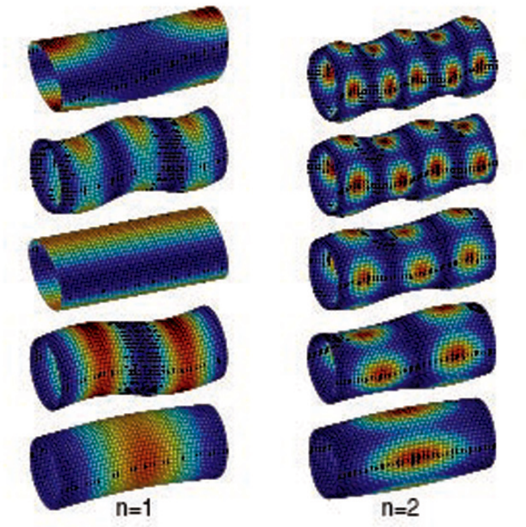

Figure 8. First five coupled mode shapes of the cylindrical shell for harmonic $n = 1$ and $n = 2$ in the open circuited case.

fact that this axial mode (Figure 7) does not induce any electromechanical coupling due to the radial electric polarization. For the other modes, as expected, the natural frequencies are higher in the open-circuit case than in the closed-circuit one. This weak difference should not be neglected. It might be used to assess the piezoelectric effect through the so-called effective modal electromechanical coupling coefficient.

6 Conclusions

We have presented in this lecture appropriate variational formulations for linear vibration of elastic and piezoelectric structures coupled with an internal acoustic fluid. Moreover, a dissipative wall fluid-structure interface model has been presented and analysed. Hybrid simultaneous passive/active damping treatments is the subject of current investigations for noise and vibration reduction problems (following the analyses carried out for structural damping (Galucio et al., 2005)). For sake of brevity, we have not presented fluid-structure symmetrization techniques for the introduced variational formulations and we refer the reader to the cited specific references such as (Morand and Ohayon, 1995; Deü et al., 2008).

Bibliography

A. Bermúdez and R. Rodríguez. Finite element computation of the vibration modes of a fluid-solid system. *Computer Methods in Applied Mechanics and Engineering*, 119(3-4):355–370, 1994.

A. Bermúdez, L. Hervella-Nieto, and R. Rodríguez. Finite element computation of the vibrations of a plate-fluid system with interface damping. *Computer Methods in Applied Mechanics and Engineering*, 190(24-25): 3021–3038, 2001.

P. Davidsson and G. Sandberg. A reduction method for structure-acoustic and poroelastic-acoustic problems using interface-dependent Lanczos vectors. *Computer Methods in Applied Mechanics and Engineering*, 195 (17-18):1933–1945, 2006.

J.-F. Deü and W. Larbi. A state space method for free-vibration analysis of radially polarized laminated piezoelectric cylinder filled with fluid. In *Proceedings of the Eighth International Conference on Computational Structures Technology, CST2006*, Las Palmas de Gran Canaria, Spain, September 12-15, 2006.

J.-F. Deü, W. Larbi, and R. Ohayon. Dissipative interface modeling for vibroacoustic problems - A new symmetric formulation. In C.A. Mota Soares, J.A. Martins, H.C. Rodrigues, and J.A.C Ambrósio, editors, *Computational Mechanics – Solids, Structures and Coupled Problems*, pages 413–428, Dordrecht, Netherlands, 2006. Springer.

J.-F. Deü, W. Larbi, and R. Ohayon. Piezoelectric structural acoustic problems: Symmetric variational formulations and finite element result. *Computer Methods in Applied Mechanics and Engineering*, 197(19-20): 1715–1724, 2008.

A.C. Galucio, J.-F. Deü, and R. Ohayon. A fractional derivative viscoelastic model for hybrid active-passive damping treatments in time domain - application to sandwich beams. *Journal of Intelligent Material Systems and Structures*, 16(1):33–45, 2005.

V. Kehr-Candille and R. Ohayon. Elastoacoustic damped vibrations - finite element and modal reduction methods. In O.C. Zienkiewicz P. Ladevèze, editor, *New Advances in Computational Structural Mechanics*, pages 321–334, Amsterdam, Netherlands, 1992. Elsevier.

W. Larbi, J.-F. Deü, and R. Ohayon. A new finite element formulation for internal acoustic problems with dissipative walls. *International Journal for Numerical Methods in Engineering*, 68(3):381–399, 2006.

W. Larbi, J.-F. Deü, and R. Ohayon. Vibration of axisymmetric composite piezoelectric shells coupled with internal fluid. *International Journal for Numerical Methods in Engineering*, 71(12):1412–1435, 2007.

H.J.-P. Morand and R. Ohayon. *Fluid-Structure Interaction.* Wiley, New York, 1995.

R. Ohayon. Fluid-structure interaction problems. In E. Stein, R. de Borst, and T. Hughes, editors, *Encyclopedia of Computational Mechanics, Volume 2: Solids and Structures, Chapter 21*, Chichester, UK, 2004a. John Wiley & Sons.

R. Ohayon. Reduced models for fluid-structure interaction problems. *International Journal for Numerical Methods in Engineering*, 60(1):139–152, 2004b.

R. Ohayon and C. Soize. *Structural Acoustics and Vibration.* Academic Press, London, 1998.

K.C. Park, C.A. Felippa, and R. Ohayon. Partitioned formulation of internal fluid-structure interaction problems by localized Lagrange multipliers. *Computer Methods in Applied Mechanics and Engineering*, 190(24-25): 2989–3007, 2001.

H.F. Tiersten. *Linear piezoelectric plate vibration.* Plenum Press, New York, 1969.

Fundamentals of Fluid-Structure Interaction

Göran Sandberg*, Per-Anders Wernberg*, Peter Davidsson †
* Department of Construction Sciences, Structural Mechanics, Lund University, Lund, SWEDEN
† A2Acoustics, Helsingborg, SWEDEN

Abstract Acoustic and structure-acoustic analysis is of great importance are found in a number of applications. Common applications for acoustic and structure-acoustic analysis are the passenger compartments in automobiles and aircraft. The increased use of light-weight materials in these vehicles usually makes it even more complicated to achieve good passenger comfort in terms of low level of interior noise. When the weight of the structure is reduced, the vibrations could be increased and that could lead to higher noise levels. Another application where structure acoustic analysis is of interest is in light-weight constructions of buildings, to mention but a few.

This paper describes the basic finite element formulations of coupled fluid-structure systems and an overview of the various formulations possible. Then a scheme for treating unsymmetrical coupled systems is outlined. The discretization is performed using displacement formulation in the structure and either pressure or displacement potential in the fluid. Based on the eigenvalues of each subdomain some simple steps give a standard eigenvalue problem. It might also be concluded that the unsymmetrical matrices have real eigenvalues. The strategies for formulating the structure-acoustic systems are illustrated using the educational software routines and elements.